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Electrodynamics
Final Exam

Date: *Friday, July 1, 2016*
Duration: *3 hours*
Total Points: *25*

1. *Use a seperate sheet for every exercise.*
2. *Write your name and initials on all sheets, on the first sheet also your address and your student ID number.*
3. *Write clearly, unreadable work cannot be corrected.*
4. *You may use the book of Griffiths.*
5. *Distribute your time wisely between the exercises.*

1. Interaction of two dipoles (4 points)

A magnetic dipole $\vec{m}_1$ at $x = y = z = 0$ is oriented in the z direction.

a) A second dipole $\vec{m}_2$ is positioned at $x = y = 0$ and can move in the z direction. Calculate the force $\vec{F}(z)$ between the two dipoles and the potential energy $U(z)$, when

1. $\vec{m}_2$ points also in the z direction.
2. $\vec{m}_2$ points in the x direction.

(2 points)

b) Now the second dipole $\vec{m}_2$ is positioned at $y = z = 0$ and can move in the x direction. Calculate the force $\vec{F}(x)$ and the potential energy $U(x)$ for the same two directions of $\vec{m}_2$ as above. (2 points)

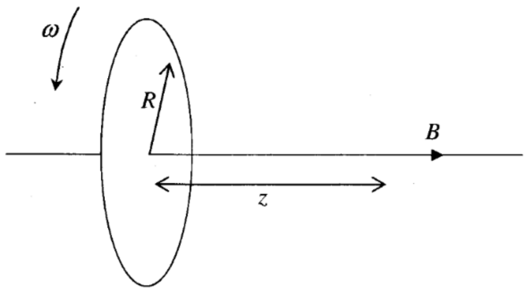
2. A charged disk as a magnetic dipole (8 points)

A thin, non-conducting disk of radius R is spinning around its symmetry axis with angular velocity ω. The disk is uniformly charged with a charge density per unit area σ.

a) Show that the exact expression for the magnetic field along the symmetry axis of the disk as a function of the distance z is given by

$$\vec{B}(z) = \frac{\mu_0 \sigma \omega}{2} \left(\frac{R^2 + 2z^2}{\sqrt{R^2 + z^2}} - 2|z| \right) \hat{z} \, ,$$

with $\hat{z}$ being the unit vector in z-direction. (Hint: $\int \frac{r^3}{(r^2+z^2)^{3/2}} dr = \frac{r^2+2z^2}{\sqrt{r^2+z^2}}$) (4 points)



b) For distances far from the disk, the disk looks like a magnetic dipole. Show by integration of the surface current density

$$\vec{K}(r_{2d}) = r_{2d}\omega\sigma\hat{\phi}$$

(r_{2d} is the distance in the x-y-plane to the origin and $\hat{\phi}$ is the unit vector in azimuthal direction), that the dipole moment is given by

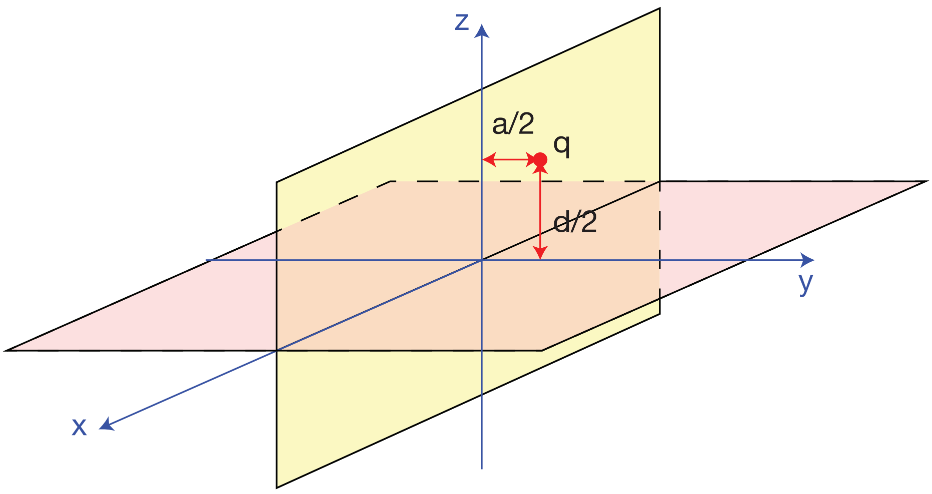
$$\vec{m} = \frac{\pi\sigma\omega R^4}{4}\hat{z} \tag{1}$$

where $\hat{z}$ is the unit vector in z-direction. (2 points)

c) Show that the expressions obtained agree at large distances from the disk (Hint: you need to perform a Taylor series up to fourth order). (2 points)

3. Test charge over two crossed grounded metal plates (7 points)

We consider a point charge $+q$ which is placed at the point $\vec{x}_1 = (0, a/2, d/2)$. There are two grounded plates, one in the xy-plane thus defined by $z = 0$, another in the xz-plane, defined by $y = 0$. Consequently, they share the x-axis. The plates are at potential $V = 0$.



a) Determine the potential in this situation using the method of image charges. Show that it is given by $V(\vec{r}) = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{|\vec{r}-\vec{x}_1|} + \frac{1}{|\vec{r}-\vec{x}_2|} - \frac{1}{|\vec{r}-\vec{x}_3|} - \frac{1}{|\vec{r}-\vec{x}_4|}\right)$ with $\vec{x}_1 = (0, a/2, d/2)$, $\vec{x}_2 = -\vec{x}_1$, $\vec{x}_3 = (0, -a/2, d/2)$, and $\vec{x}_4 = -\vec{x}_3$ by explicitly showing that the boundary condition is fulfilled.(1 point)

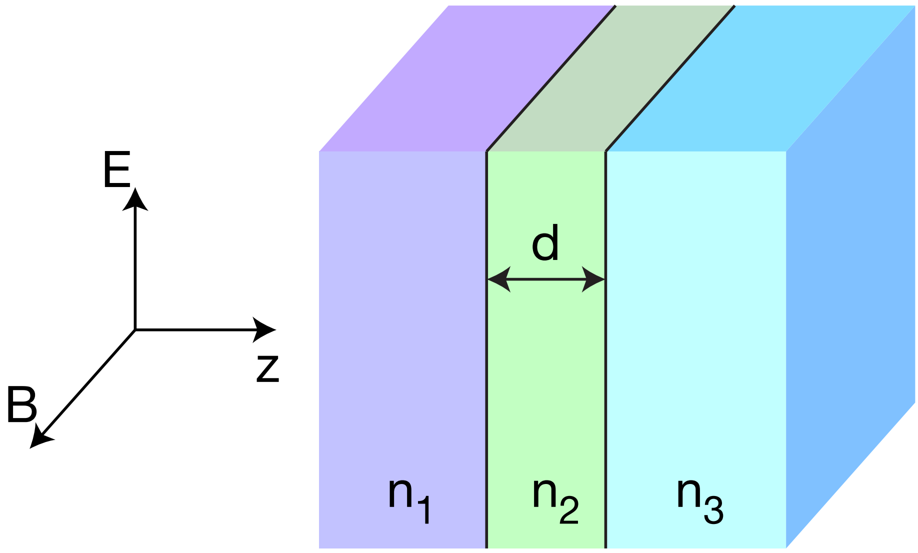
b) Determine the charge distribution on the plates from the potential given in **a)**. (2 points)

c) For this part of the exercise assume $a = d$ in the potential in **a)**. Use Taylor expansion for $r \gg d$ to show that the dipole moment vanishes. Argue whether the quadrupole moment is finite. (2 points)

d) Now we assume that $a \gg d$. We now shift the coordinate system such that the new origin sits at $(0, a/2, 0)$ and positions are now given with respect to this new origin. In terms of this position $\vec{r}$ there are now three regimes in which the potential approximately looks like (a) a monopole, (b) a dipole, and (c) a quadrupole. Make a sketch and draw these regions and give a short justification. (2 points)

4. Transmission of light (6 points)

A plane wave is incident normally on a layered interface as shown in the figure from the left in z-direction. The indices of refraction of the three non-permeable media are n_1, n_2, and n_3. The intermediate layer is located between $z = 0$ and $z = d$. Each of the other media is semi-infinite. One can make an ansatz in the three regions: on the left (region with n_1) we



have $\vec{E}_I = \vec{E}_0 e^{i\vec{k}_1\cdot\vec{r}-i\omega t} + \vec{E}_1 e^{-i\vec{k}_1\cdot\vec{r}-i\omega t}$, in the intermediate region (with n_2) we have $\vec{E}_{II} = \vec{E}_{2+} e^{i\vec{k}_2\cdot\vec{r}-i\omega t} + \vec{E}_{2-} e^{-i\vec{k}_2\cdot\vec{r}-i\omega t}$ and on the right (with n_3) we have the transmitted one given by $\vec{E}_{III} = \vec{E}_3 e^{i\vec{k}_3\cdot\vec{r}-i\omega t}$.

a) Derive the boundary conditions for the two interfaces. (4 points)

b) Show that the reflection coefficient is given by

$$R = \left| \frac{\left(1 - \frac{n_2}{n_1}\right)\left(1 + \frac{n_3}{n_2}\right) + \left(1 + \frac{n_2}{n_1}\right)\left(1 - \frac{n_3}{n_2}\right) e^{2ik_2 d}}{\left(1 + \frac{n_2}{n_1}\right)\left(1 + \frac{n_3}{n_2}\right) + \left(1 - \frac{n_2}{n_1}\right)\left(1 - \frac{n_3}{n_2}\right) e^{2ik_2 d}} \right|^2 .$$

(2 points)