FULL EXAM GEOPHYSICAL FLUID DYNAMICS

3 February 2010, 9.00 - 12.00 hours

Four problems (all items have equal weight)

Remark 1: answers may be written in English or Dutch.

Remark 2: in all questions you may use $g = 10\ \mathrm{ms}^{-2}$, $a = 6400$ km and $\Omega = 7.3\times 10^{-5}\ \mathrm{s}^{-1}$.

Problem 1

Consider the zonal momentum balance for a fluid on the rotating earth,

$$\rho\left(\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} + w\,\frac{\partial u}{\partial z} + f_*\,w - f\,v\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

a. Explain the meaning of the first four terms on the left-hand side of this equation. Limit your answer to 0.25 A4 maximum.

b. Discuss the Boussinesq approximation and show that its application to the equation above results in

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(uu) + \frac{\partial}{\partial y}(uv) + \frac{\partial}{\partial z}(uw) + f_*\,w - f\,v = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

where p is now the dynamic pressure.
Give also the definition of ρ_0, ν and of the dynamic pressure.

c. Specify the condition(s) under which the flow described by the equation above will be turbulent.
Are geophysical flows often turbulent?

d. Apply a Reynolds averaging procedure to the equation given in item b.
Discuss the main steps of the procedure, present the equation for the resolved flow and indicate where the Reynolds stresses appear in the final equation.

For problem 2: P.T.O.

Problem 2

Consider a zonal flow on the Southern Hemisphere, given by

$$u = \begin{cases} -U & \text{if} \quad y > L\,, \\ -U\,y/L & \text{if} \quad -L \le y \le L\,, \\ U & \text{if} \quad y < -L\,, \end{cases}$$

where $U > 0$.

a. Assuming that this flow is geostrophic, and that the f-plane approximation holds, compute the pressure distribution that maintains the flow.
Assume that the pressure has a fixed value $\hat{p}$ at $y = 0$.

b. Compute the absolute vorticity distribution of this flow.

c. Is the relative circulation in a square box, with sides $2\,L$ and with its centre at $y = 0$, cyclonic, anticyclonic or both?
Motivate your answer.

d. Compute and sketch the distribution of the Ekman pumping that is induced by friction near the bottom.

For problem 3: next page

Problem 3

Consider the following equations:

$$
\begin{aligned}
\left(\frac{\partial}{\partial t} + U_1 \frac{\partial}{\partial x}\right) \left[\nabla^2 \psi_1' + \frac{1}{2R^2}\left(\psi_2' - \psi_1'\right)\right] + \left(\beta_0 + \frac{\Delta U}{2R^2}\right) \frac{\partial \psi_1'}{\partial x} &= 0\,, \\
\left(\frac{\partial}{\partial t} + U_2 \frac{\partial}{\partial x}\right) \left[\nabla^2 \psi_2' - \frac{1}{2R^2}\left(\psi_2' - \psi_1'\right)\right] + \left(\beta_0 - \frac{\Delta U}{2R^2}\right) \frac{\partial \psi_2'}{\partial x} &= 0\,.
\end{aligned}
$$

a. Name and describe the meaning of the variables ψ_1', ψ_2' , as well as the parameters U_1, U_2, R, β_0 and ΔU.

b. Show that these equations admit wave-like solutions, and derive two equations for the complex constants ϕ_1 and ϕ_2 that appear in these wave-like solutions.

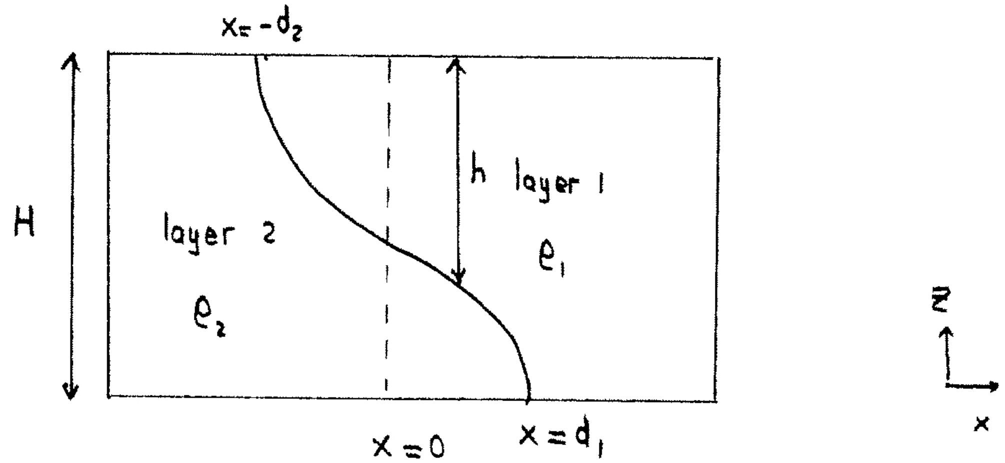
c. Indicate how <u>in principle</u> the dispersion relation of the waves can be derived from the two equations of item b.
(no derivation of the dispersion relation is required).

d. Under certain conditions wave-like solutions of the two equations given above are found, of which the amplitude increases exponentially in time.
Name this mechanism, discuss which parameters control this mechanism and whether an increase of each parameter causes larger or smaller instability.
Limit your answer to at most 0.5 A4.

For problem 4: P.T.O.

Problem 4

Two layers of fluids, with densities ρ_1 and ρ_2, are initially separated by a vertical wall at $x = 0$ (the dashed line in the figure below).



At time $t = 0$ the vertical wall is removed and the system adjusts to a final steady state. In the figure, the interface between the two fluids in the end state is indicated by the solid curve (from $x = -d_2$ to $x = d_1$). Note that thickness $h = H - a$. The dynamics is governed by the nonlinear, frictionless shallow water equations for a two-layer system on the f-plane.

a. Considering the situation sketch above, is density ρ_1 larger or smaller than ρ_2? Motivate your answer.

b. From analysing the equations of motion, it follows that

$$\frac{dv_1}{dx} = f\left[\frac{h}{H} - 1\right], \qquad \frac{dv_2}{dx} = -f\frac{h}{H},$$

where v_1, v_2 are the velocities in the layers with densities ρ_1, ρ_2. Describe briefly how these results are obtained, and define the principle that is crucial in this respect.

c. A third equation relating v_1, v_2 and h in the end state is

$$f\,(v_1 - v_2) = g'\,\frac{dh}{dx}.$$

Name this balance, and explain how it is derived from the given equations of motion.

d. From the equations of items b and c the thickness $h(x)$ of the end state can be found, using the boundary conditions $h(x = -d_2) = 0$ and $h(x = d_1) = H$. In order to determine the locations d_1 and d_2 two additional constraints are needed. Describe these constraints, both physically and in terms of mathematical expressions.

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