MID-TERM EXAM GEOPHYSICAL FLUID DYNAMICS

10 November 2010, 9.00 - 11.00 hours

Two problems (all items have equal weight)

Remark: answers may be written in English or Dutch.

Problem 1

Consider a fluid that is bounded from below by a fixed level $z = b$ and from above by a free level $z = \eta$.
One of the equations governing the dynamics of this fluid is

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} - f\,u = -g\,\frac{\partial \eta}{\partial y} \qquad (*)$$

a. Name, define and describe the physical meaning of parameter f.
Also, name the variables that appear in the equation f.

b. Is this flow characterised by a small Richardson number?
Explain your answer by defining this number and explaining its physical meaning.

c. Name the first term on the right-hand side of Eq. (*) and derive it from a term that occurs in the equations of motion for a molecular viscous fluid.

Assume that the flow is steady, that $\partial b/\partial x = 0$ and that $\partial b/\partial y = 0.02$. Furthermore, it is given that in point P, which is at location $x = 0, y = 0$ and halfway the fluid column, velocity $v = 1$ m s^{-1} and velocity $w = 0.05$ m s^{-1} (see figure).

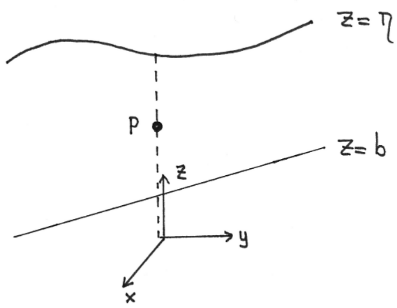


Figure 1: Situation sketch. The x-axis points out of the paper.

d. Compute at this x, y-location the values of v and w at the bottom.
Explain how you obtain your answer.

e. Compute at this x, y-location the values of v and w at the free surface.
Explain how you obtain your answer.

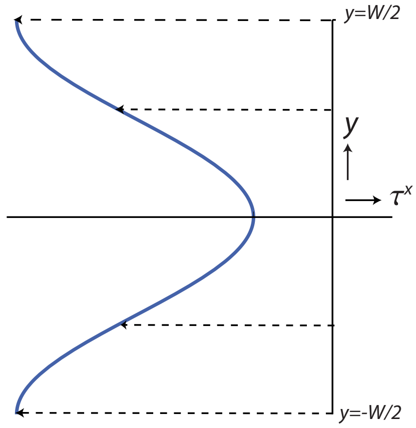
For problem 2: P.T.O.

Problem 2

In an area north of the equator the wind stress over the surface of the ocean is given by

$$\tau^x = -T_0 + \hat{T} \cos\left(\frac{2\pi y}{W}\right), \qquad \tau^y = 0.$$

Here, $-W/2 \leq y \leq W/2$ and $0 < \hat{T} < T_0$ (see figure).
This winds stress generates steady and linear large-scale flow that is uniform in the zonal direction. Furthermore, assume parameters f and ν_E to be constants.



a. Assuming that there is no geostrophic flow in the interior of the ocean (the ocean is infinitely deep), sketch the velocity vector (with components u and v) in the ocean as a function of vertical coordinate z at location $y = 0$.
Explain your answer.

b. Compute the transports U and V at $y = 0$ in terms of parameters $f, T_0, \hat{T}$ and W.
Hint: integrate the momentum equations.

c. Compute and sketch the distribution of the surface Ekman pumping velocity as a function of y.
Also, give a short interpretation of your result.

Assume now that the ocean has a finite depth and that the Ekman pumping near the bottom is identical to that induced by the surface Ekman layer.

d. Sketch the resulting pressure distribution in the ocean and the transport distribution in the bottom Ekman layer.

e. Compute the geostrophic flow in the interior of the ocean, assuming that its area-averaged value is zero.

f. Derive an expression for the (relative) circulation in the interior of the ocean in the domain defined by $0 \leq x \leq L$ and $0 \leq y \leq W/2$. Here, L is a fixed length.

END

GFD 2010 Equation sheet

Continuity and momentum equations: molecular viscous fluid

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

$$\begin{aligned}
\rho\left(\frac{du}{dt} + f_* w - f v\right) &= -\frac{\partial p}{\partial x} & &+\mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \\
\rho\left(\frac{dv}{dt} + f u\right) &= -\frac{\partial p}{\partial y} & &+\mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) \\
\rho\left(\frac{dw}{dt} - f_* u\right) &= -\frac{\partial p}{\partial z} - \rho g & &+\mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)
\end{aligned}$$

Energy budget for adiabatic flow of fixed composition

$$\rho C_v \frac{dT}{dt} - \frac{T}{\rho}\left(\frac{\partial p}{\partial T}\right)_\rho \frac{d\rho}{dt} = 0$$

Relative circulation and relative vorticity

$$\Gamma = \oint_C \mathbf{u} \cdot d\mathbf{r} = \int_S (\nabla \times \mathbf{u}) \cdot \mathbf{n}\, dS$$

where S is the surface enclosed by contour C.

Shallow water equations

$$\begin{aligned}
\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - f v &= -g\frac{\partial \eta}{\partial x} \\
\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + f u &= -g\frac{\partial \eta}{\partial y} \\
\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) &= 0
\end{aligned}$$

Ekman pump (Northern Hemisphere)

$$\overline{w} = \frac{d}{2}\overline{\zeta}, \qquad d = \left(\frac{2\nu_E}{f}\right)^{1/2} \qquad \text{and} \qquad w_{\mathrm{E}k} = \frac{1}{\rho_0 f}\left[\frac{\partial \tau^y}{\partial x} - \frac{\partial \tau^x}{\partial y}\right]$$