MID-TERM EXAM GEOPHYSICAL FLUID DYNAMICS

10 October 2016, 9.00 - 11.00 (2 hours)

Three problems, total 45 points.

Please write your answers for each problem on a separate sheet of paper and put your name and student number on each sheet.

Answers may be written in English or Dutch. Please write clearly!

In all questions you may use $g = 10$ ms^{-2}, $r_a = 6400$ km and $\Omega = 7.3 \times 10^{-5}$ s^{-1}.

Problem 1

Consider the continuity equation and momentum equations for a molecular viscous fluid on a rotating Earth, as are given on the supplementary equation sheet.

a. Are these equations exact?
If not, what simplifications have been made? (3 points)

b. Specify the Boussinesq approximation and use it to derive a simpler version of the mass budget (4 points).

√c. Write down the momentum budget in the south-north direction after application of the Boussinesq approximation and Reynolds averaging procedure.
What are the similarities and differences with respect to the momentum budget for a molecular viscous fluid? (4 points)

√ d. Under what condition(s) will the vertical momentum budget reduce to hydrostatic balance?
Are there additional terms in the continuity and momentum equations that can be ignored in that case? If so, which one(s)? (4 points)

Problem 2

Between 15°N and 45°N, the winds over the North Pacific Ocean consist mostly of the easterly trades (15°N to 30°N) and the midlatitude westerlies (30°N to 45°N). An adequate representation is

$$\tau^x = \tau_0 \sin\left(\frac{\pi y}{2L}\right), \quad \tau^y = 0 \quad \text{for} \quad -\mathrm{L} \le \mathrm{y} \le \mathrm{L},$$

with $\tau_0 = 0.15$ N m^{-2} (maximum wind stress) and L = 1670 km.

√ a. Give the equations of motion for the Ekman layer and explain the assumptions made (3 points).

√ b. Taking $\rho_0 = 1028$ kg m^{-3} and the value of the Coriolis parameter corresponding to 30°N, calculate the Ekman pumping. Which way is it directed? Would you expect high or low biological productivity in this region of the ocean? (4 points)

For problems 2.c-d and 3.: P.T.O.

c. Calculate the vertical volume flux over the entire 15°N - 45°N strip of the North Pacific (width = 8700 km). Express your answer in sverdrup units (1 sverdrup = 1 Sv = 10^6 m^3 s^{-1}). (4 points)

d. Calculate the thickness of the surface Ekman layer at 15°N, 30°N and 45°N with a vertical eddy viscosity $\nu_E = 0.1 m^2 s^{-1}$. Where do you expect the thickest Ekman layer? (4 points)

Problem 3

Consider the meridional momentum balance for a frictionless, homogenous, rotating fluid:

$$\frac{dv}{dt} + f u = -\frac{1}{\rho_0}\frac{\partial p}{\partial y}. \qquad (*)$$

and assume barotropic flow, i.e., u and v independent of the vertical coordinate.

a. Assume that the fluid is bounded from above by a free surface $z = \eta$ and at the bottom by a fixed surface $z = b$.
Specify the boundary conditions at $z = \eta$ and $z = b$ that correspond to equation (*). Use the boundary condition(s) at $z = \eta$ to rewrite the pressure in equation (*). (4 points) $p = p_a + \rho_0 g \eta$

b. Use the other boundary conditions to formulate an alternative equation for the vertically integrated continuity equation (under Boussinesq approximation). (3 points)

c. The meridional momentum balance for a Kelvin wave travelling along a coast located at $x = 0$ reads

$$\frac{\partial v}{\partial t} = -g\frac{\partial \eta}{\partial y}.$$

How, and under what conditions for key dimensionless numbers that characterise geophysical fluids, is this equation derived from equation (*)? (4 points)

d. Sketch the structure of the free surface and the velocity components u, v for a Kelvin wave that travels along a coast that is located at $x = 0$ in the Southern Hemisphere.
Pay attention to the direction of propagation of the wave and its cross-shore structure (4 points).

END

GFD 2016 Equation sheet

Continuity and momentum equations: molecular viscous fluid

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho\, u) + \frac{\partial}{\partial y}(\rho\, v) + \frac{\partial}{\partial z}(\rho\, w) = 0$$

$$\begin{aligned}
\rho\left(\frac{du}{dt} + f_* w - f\,v\right) &= -\frac{\partial p}{\partial x} & &+ \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)\\
\rho\left(\frac{dv}{dt} + f\,u\right) &= -\frac{\partial p}{\partial y} & &+ \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)\\
\rho\left(\frac{dw}{dt} - f_* u\right) &= -\frac{\partial p}{\partial z} - \rho g & &+ \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)
\end{aligned}$$

Relative circulation and relative vorticity

$$\Gamma = \oint_C \mathbf{u}\cdot d\mathbf{r} = \int_S (\nabla \times \mathbf{u})\cdot \mathbf{n}\, dS$$

where S is the surface enclosed by contour C.

Shallow water equations

$$\begin{aligned}
\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - f\,v &= -g\frac{\partial \eta}{\partial x}\\
\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + f\,u &= -g\frac{\partial \eta}{\partial y}\\
\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) &= 0
\end{aligned}$$

Ekman pump (Northern Hemisphere)

$$\overline{w} = \frac{d}{2}\overline{\zeta}, \qquad d = \left(\frac{2\nu_E}{f}\right)^{1/2} \qquad \text{and} \qquad w_{\mathrm{Ek}} = \frac{1}{\rho_0\, f}\left[\frac{\partial \tau^y}{\partial x} - \frac{\partial \tau^x}{\partial y}\right]$$