DEPARTMENT OF PHYSICS AND ASTRONOMY, FACULTY OF SCIENCE, UU.
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IN 2005/2006, THE COURSE NS-353B WAS GIVEN BY P.J. VAN LEEUWEN.

Geophysical Fluid Dynamics (NS-353B)
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Question 1

We study a cyclone on the northern hemisphere in geostrophic balance. Its pressure field is given by:

$$p = -p_0 \exp\left[-\frac{x^2 + y^2}{2L^2}\right]$$

in which $L = 1000$ km. The density is constant $\rho = \rho_0$.

a) Discuss the conditions that lead to geostrophic balance, starting from the zonal momentum equation given by

$$u_t + uu_x + vu_y + wu_z - fv = -\frac{1}{\rho_0}p_x + Au_{zz}$$

b) Calculate u and v, assuming $f = f_0$.

c) Calculate the relative vorticity ζ, and sketch its meridional profile through the cyclone center.

d) Choose $f = f_0 + \beta y$ and recalculate ζ.

e) Determine the meridional distance between the maxima of p and ζ, using $\beta = 2\ 10^{-11}\ m^{-1}s^{-1}$ and $f_0 = 10^{-4}\ s^{-1}$. Hint: use only terms to first order in y/L when calculating the position of the maximum of ζ. Note that also $\beta L/f_0 \ll 1$.

Question 2

The cyclone from exercise 1 loses energy due to friction at the bottom. We assume that Ekman friction is a reasonable description.

a) Derive the vorticity equation from the momentum equations in isobaric coordinates, given by:

$$\begin{aligned} u_t - f_0 v &= -\phi_x \\ v_t + f_0 u &= -\phi_y \end{aligned}$$

b) Use the continuity equation to rewrite the vorticity equation in the form:

$$\zeta_t = f_0 w_z$$

Explain the meaning of this equation.

c) Integrate this equation over the geostrophic interior, assuming a vanishing vertical velocity at the top of the layer. Use the expression for the vertical velocity at the top of the Ekman layer to find

$$\zeta_t = -\frac{f_0 d}{2H}\zeta$$

in which H is the thickness of the interior layer, and d is the Ekman-layer thickness.

d) Solve this equation, and determine the spin-down time of the cyclone, given $f_0 = 10^{-4}\ s^{-1}$, $d = 100$ m, and $H = 10$ km.

e) Explain why the cyclone spins down using a vorticity argument.

Question 3

We study Rossby-wave propagation in a barotropic fluid. The quasi-geostrophic (QG) potential vorticity equation reads:

$$\frac{\mathrm{d}q}{\mathrm{d}t} = 0$$

in which the potential vorticity is given by

$$q = \Delta\phi + f_0 + \beta y - \frac{1}{R_d^2}\psi$$

with the external Rossby radius of deformation given by

$$R_d = \frac{\sqrt{gH}}{f_0}$$

a) Explain the meaning of the different terms in the expression for the potential vorticity.

b) Linearize the QG potential vorticity equation around a state of rest.

c) Determine the dispersion relation of plane waves of the form

$$\phi = A\exp[i(kx + ly - \omega t)]$$

d) Show that waves with an eastward energy-transport component have to fulfill

$$k^2 > l^2 + \frac{1}{R_d^2}$$

e) Determine the maximum angular frequency of purely zonal Rossby waves.

f) What is the physical meaning of this maximum angular frequency?

Question 4

Consider a steady current in a two-layer fluid flowing along the eastward side of a meridional coastline. Use $f = f_0 = 10^{-4}\ \ s^{-1}, \rho_1 = 1024\ kgm^{-3}$, and $\rho_2 = 1026\ kgm^{-3}$. The undisturbed layer thicknesses are $H_1 = 500\ m$, and $H_2 = 1500\ m$.

a) Show that when a steady current flows parallel to such a coast, and friction is neglected, it has to be in geostrophic balance.

The upper and lower layer velocity fields are given by

$$v_1 = V_1\frac{L-x}{L} \qquad \text{for} \qquad 0 \le x \le L$$
$$v_2 = V_2\frac{L-x}{L} \qquad \text{for} \qquad 0 \le x \le L$$

with $V_1 = 1\ ms^{-1}, V_2 = 0.2\ ms^{-1}$, and $L = 20\ km$.

b) Determine the surface elevation $\xi(x)$ from geostrophy.

c) Determine the interface elevation $\eta(x)$ from geostrophy. Use that the pressure in the second layer is given by $p_2 = g\xi - g'\eta$, in which η is measured positive downward.

d) Calculate the transport, in m^3s^{-1}, in the upper and in the lower layer, neglecting the surface elevation.

The current encounters a bottom escarpment such that $H_2 = 1250\ m$.

e) Determine the new velocity profile in the lower layer assuming that it keeps its triangular shape (so determine the new V_2 and the new L). Neglect surface and interface variations. Hint: use two conserved quantities.