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In 2004/2005, the course NS-353b was given by P.J. van Leeuwen.

Geofysische Stromingsleer (NS-353b)
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Question 1

Consider a cyclonic vortex in geostrophic balance.

a) Which forces balance in geostrophic balance? Sketch this balance, and the velocity vector on the northern hemisphere.

b) Sketch the circulation around a cyclone on the northern hemisphere. Explain why a cyclone can live for several days before *filling up*.

c) One of the forces that is neglected in the geostrophic balance is the centrifugal force. Will the velocity in the cyclone be smaller or larger when this force is taken into account (for the same pressure distribution)?

d) Repeat c) for an anticyclonic high-pressure vortex.

e) Can an anticyclonic rotating low-pressure vortex exist? Explain your answer.

The zonal momentum balance through the center of the vortex including the centrifugal force reads:

$$-fv - \frac{v^2}{x} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

f) show that a low-pressure vortex can have more extreme pressure anomalies than a high-pressure vortex. (Hint: Look at the solution for the meridional velocity)

Question 2

Consider a fluid column in a fluid of homogeneous density with negligible friction, on a β-plane.

a) Give an expression for potential vorticity conservation. Explain this expression using Kelvin's theorem.

b) Explain that vortex stretching without meridional motion leads to strengthening of a cyclonic vortex and weakening of an anticyclonic vortex, independent of the hemisphere.

c) Explain that pole-ward motion will weaken a cyclone and strengthen an anticyclone when stretching is negligible.

Assume that layer-thickness variations are due to bottom topography changes.

d) Show that when the bottom topography changes δh are small relative to the mean depth H the potential vorticity conservation can be formulated as

$$(\zeta + f)\left(1 - \frac{\delta h}{H}\right) = \text{constant}$$

following the parcels.

e) Assume that the products $\zeta\frac{\delta h}{H}$ and $\beta y\frac{\delta h}{H}$ can be neglected. For what bottom topography does the potential vorticity conservation reduce to $\zeta =$ constant?

Question 3

We study the fluid circulation betwee two rotating disks 10 cm apart. The disks are so large that effects from their edges can be neglected. The density of the fluid is constant in space and time. The disks rotate so fast that the planetary vorticity can be neglected. The lower disk rotates with vorticity $\omega_B = \Omega = 0.2s^{-1}$, and the upper with $\omega_T = \Omega(1+\delta)$ in which $0 \leq \delta \ll 1$. Ω is so large that the interior fluid moves in Taylor columns.

a) What is the Taylor-Proudman theorem?

Close to the disks Ekman layers exist.

b) Calculate the Ekman-layer thickness d when the kinematic velocity $A_v = 10^{-5}m^2s^{-1}$.

c) Argue that the vertical velocity at the top of the lower Ekman layer is given by

$$w = \frac{d}{2}(\omega_I - \omega_B)$$

in which ω_I is the velocity in the interior.

d) Derive a similar expression for the vertical velocity at the interior side of the upper Ekman layer.

e) Use c) and d) and the Taylor-Proudman theorem to express ω_I in terms of Ω and δ.

f) Sketch the fluid motion between the disks connected to these vertical Ekman velocities.

Assume now that $\delta = 0$. The disks are accelerated so that their vorticity changes from Ω tot $\Omega + \epsilon$ with $0 \leq \epsilon \ll \Omega$

g) Sketch the fluid motion during this spin up.

h) Explain the spin up of the interior fluid in terms of potential vorticity conservation in the interior.