EXAM CLASSICAL FIELD THEORY

June 28, 2011

- The duration of the test is 3 hours.
- The lecture notes by Gleb Arutynov "Classical Field Theory" and the book by Lev Landau and Evgeny Lifshitz "Field Theory" may be consulted during the test.
- Usage of a calculator and a dictionary is allowed

Problem 1

Consider the following action for a complex scalar field $\psi(x,t)$ in two space-time dimensions

$$S = \int \mathrm{d}x\mathrm{d}t \left(i\bar{\psi}\dot{\psi} - m^2 \partial_x \bar{\psi} \partial_x \psi - 2\kappa \left(|\psi|^2 \right)^2 \right)$$

Here m^2, κ are parameters and $\bar{\psi}$ is the complex conjugate of ψ.

1. Argue that this action in invariant under global transformations $\psi \to e^{i\alpha}\psi$, where $\alpha \in \mathbf{R}$ is a transformation parameter;
2. Derive the Noether current corresponding to these symmetry transformations;

Problem 2

Consider an electric field of the following profile

$$\vec{E} = q\frac{\vec{r}}{r^3}\left(1 + br\right) e^{-br},$$

where q and b are some positive constants and r is a distance to the origin of the coordinate system.

1. Find the electric density distribution ρ corresponding to this electric field;
2. Find the total charge Q.

Problem 3

Lorentz transformations of electric and magnetic fields form a stationary frame to the one moving with velocity $\vec{v}$ have the following form

$$\begin{aligned} \vec{E}' &= a\vec{E} - \frac{a-1}{v^2}\vec{v}\left(\vec{v}\cdot\vec{E}\right) + \frac{a}{c}\left(\vec{v}\times\vec{H}\right), \\ \vec{H}' &= a\vec{H} - \frac{a-1}{v^2}\vec{v}\left(\vec{v}\cdot\vec{H}\right) - \frac{a}{c}\left(\vec{v}\times\vec{H}\right), \end{aligned}$$

where $a = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$.Show by explicit computation that $\left(\vec{E}\cdot\vec{H}\right)$is an invariant of these transformations.

Problem 4

Consider the following vector and scalar potentials

$$\vec{A}(x,t) = \vec{A_0} e^{i\left(\vec{k}\cdot\vec{x} - \omega t\right)}, \quad \varphi(x,t) = 0,$$

where $\vec{A_0}$ and $\vec{k}$ are constant three-dimensional vectors, and ω is a constant frequency.

1. Derive the electric and magnetic fields corresponding to these potentials;
2. Determine the conditions imposed on $\vec{A_0}$,$\vec{k}$ and ω by Maxwell's equations assuming the absence of charge and current densities, i.e. $\rho = 0$ and $\vec{j} = 0$.