1a

Instituut voor Theoretische Fysica, Universiteit Utrecht

MID EXAM ON CLASSICAL FIELD THEORY

Tuesday, April 14, 2009

- The duration of the test is 3 hours.
- Use different sheets for each exercise.
- Write your name and initials on every sheet handed in.
- Write clearly. Unreadable text cannot be judged.
- Some problems require some calculations. Divide your available time wisely over the problems.
- During the test you are allowed to only consult the lectures "Classical Field Theory" by Dr. Gleb Arutyunov!

Problem 1

Consider the following action for n scalar fields $\phi^a(x)$, $a = 1, \dots, n$ defined on a four-dimensional Minkowski space:

$$S = \frac{1}{2} \int \mathrm{d}^4x \; \partial_\mu \phi^a \partial^\mu \phi^a \,.$$

Show that the action is invariant under the transformations $\phi^a \to O^a_b \phi^b$, where O is a constant (independent of x) orthogonal matrix. Show that for infinitezimal transformations the matrix O can be written as $O = \mathbb{1} + A$, where A is an anti-symmetric matrix. Find the conserved Noether currents corresponding to these symmetry transformations.

Problem 2

Consider a dynamical system with two degrees of freedom (x, y) and the following Hamiltonian

$$H = \frac{1}{2}p_x^2 + \frac{1}{2}\left(p_y - \frac{x}{y}\right)^2 - \frac{1}{2}\left(\frac{x}{y}\right)^2 ,$$

where p_x and p_y are momenta canonically conjugate to x and y, respectively. Find the value of the parameter α under which the expression

$$I(\alpha) = p_x + \alpha \ln\left(\frac{p_y}{y}\right)$$

becomes an integral of motion for this Hamiltonian system.

Problem 3 *Lorentz transformations*

- Show that the Lorentz transformations form a group.
- How many independent parameters are needed to specify a generic group element?
- Do improper Lorentz transformations form a subgroup of the Lorentz group?
- Describe the action of parity and time-reversal operations.

Problem 4

Consider the action for a relativistic massive particle

$$S = -mc^2 \int_{t_1}^{t_2} \mathrm{d}t \sqrt{1 - \frac{v^2}{c^2}} \,.$$

Find the corresponding Hamiltonian.

Problem 5

The value of the time-averaged potential for a neutral hydrogen atom is given by

$$\varphi(r) = e\frac{\exp(-\alpha r)}{r}\left(1 + \frac{\alpha r}{2}\right).$$

Here e is the charge of the electron and $\alpha = 2/a_0$, where $a_0 = e^2/mc^2$ is the classical radius of the electron. By using the Laplacian in spherical coordinates (you may consult the lecture notes), find the charge distribution density and explain its physical meaning.

Be careful with the singularity of the scalar potential at the origin $r = 0$.