Exam: Dynamical Meteorology
Date: November, 7, 2014, 14:00-17:00

In this exam all symbols have their normal definitions.
Answers may be given in either English or Dutch.

Problem 1 (1.5 point)

Adjustment to heating

Figure 1 shows the pattern of isobars (lines of equal pressure) at two constant height levels (constant with respect to sea level) in the atmosphere, associated with heating of the atmosphere between these two levels. In this example the heating is due to release of latent heat in clouds over the surface of small islands. Air motion is upward over land and downward over sea. The Coriolis effect is not taken into account in this picture.

(a) Explain why the pressure is relatively high at the upper level over land compared to over sea.

(b) Explain why the pressure is relatively low at the lower level over land compared to over sea.

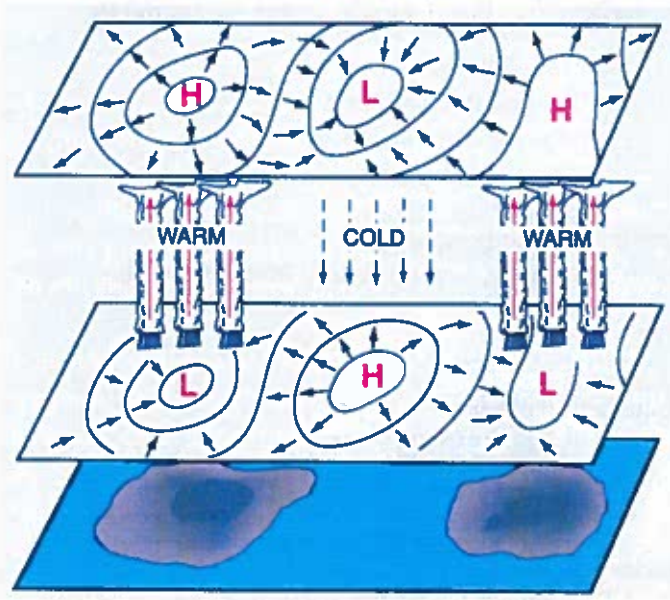

Figure 1. See problem 1.

Problem 2 (2 points)

Dew point lapse rate

The Clausius-Clapeyron equation can be written in terms of the dew point temperature, T_d, as

$$\frac{de}{dT_d} = \frac{Le}{R_v T_d^2} .$$

Here e is the water vapour pressure, R_v is the specific gas constant for water vapour and the latent heat of condensation, L, is assumed constant ($=2.5\times10^6$ J kg^{-1}). Assume that both water vapour and dry air are an ideal gas, which means that, for instance, $p_a=\rho_a R_a T$, where R_a is the specific gas constant for dry air, ρ_a is the density of dry air and p_a is the pressure exerted by the dry air molecules.

(a) Assume that $e \ll p_a$ and furthermore that the atmosphere remains very close to hydrostatic balance ($dp/dz = -\rho g$). Show that the dew point lapse rate can be approximated by

$$\frac{dT_d}{dz} \approx -\frac{gR_v T_d}{LR_a}$$

(R_a=287 J K^{-1}kg^{-1} and R_v=461.5 J K^{-1}kg^{-1}).
(b) The dry adiabatic lapse rate is $-g/c_p$ (=9.76 K km^{-1}). What criterion does L have to satisfy so that clouds will form in ascending currents of air?
(c) Is this criterion satisfied in Earth's atmosphere?

Problem 3 (2.5 points)

Rossby waves

The equation governing the perturbations (indicated by a prime) of the relative vorticity, relative to a uniform (constant) eastward geostrophic current with velocity, $\overline{U}$, in a barotropic fluid is (in pressure coordinates)

$$\frac{\partial \zeta'}{\partial t} + \overline{U}\left(\frac{\partial \zeta'}{\partial x}\right)_p = -f\left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}\right)_p - \beta v'.$$

The subscript p indicates that the derivatives are performed with pressure held constant. Assume that motion is purely horizontal on pressure surfaces (i.e. that dp/dt=0), so that the continuity equation in pressure coordinates becomes

$$\left(\frac{\partial u}{\partial x}\right)_p + \left(\frac{\partial v}{\partial y}\right)_p = 0.$$

(a) Define a streamfunction, ψ, that satisfies this continuity equation.
(b) Write the vorticity equation (see above) in terms of the streamfunction (remember that

$$\zeta = \partial v/\partial x - \partial u/\partial y.$$

(c) Assume a wave-like solution on an infinite *beta*-plane, i.e.

$$\psi' = \mathrm{Re}\left\{\Psi \exp\left[i(lx + my - \omega t)\right]\right\}$$

and derive a dispersion relation for these waves.
(d) What is the (approximate) difference (in m s^{-1}) between the eastward component of the phase speed and the eastward component of the group speed when both the zonal wavelength and the meridional wavelength of the wave is 6000 km, assuming that $\beta = 2\times10^{-11}$ m^{-1} s^{-1}.

Problem 4 (1.5 points)

Convective available potential energy

The vertical component of the acceleration of an air parcel in the atmosphere is governed *approximately* by

$$\frac{d^2z}{dt^2} \approx g\frac{\theta - \theta_0}{\theta_0} \equiv gB,$$

where θ_0 is the potential temperature in the environment of the air parcel and θ is the potential temperature of the air parcel.
(a) What approximations have been made in deriving the above equation?
(b) Suppose that the buoyancy, B, of an ascending air parcel depends on height, z, as

$$B = 0.01 \times \sin\left(\frac{\pi z}{H}\right) \text{ for } 0 \le z \le H;\ B = 0 \text{ for } z > H.$$

Compute the convective available potential energy (*CAPE*) and the maximum vertical velocity of this parcel if it starts its ascent from the Earth's surface (z=0). Assume that H=10 km.

Problem 5 (2.5 points)

Multiple choice

Indicate the "best" answer

(1) The continuity equation with pressure as a vertical coordinate is

$$-\frac{v\tan\phi}{a}+\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)_p+\frac{\partial \omega}{\partial p}=0$$

the derivation of this equations assumes
(a) incompressibility and hydrostatic balance
(b) hydrostatic balance
(c) an adiabatic atmosphere

(2) The inertial frequency increases if
(a) absolute vorticity decreases
(b) relative vorticity increases
(c) the vertical gradient of potential temperature increases

(3) During a major sudden stratospheric warming
(a) polar cap temperatures in the stratosphere increase by more than 25 K within a week
(b) polar cap temperatures in the stratosphere increase by more than 25 K within a day
(c) extra-tropical temperatures in the stratosphere increase by 25 K within a week

(4) In the Quasi-Biennial Oscilltion (QBO)
(a) the eastward wind component is stronger than the westward wind component
(b) the eastward wind component is as strong as the westward wind component
(c) the westward wind component is stronger than the eastward wind component

(5) The specific humidity is defined as
(a) the mass density of water vapour as a fraction of the mass density of air
(b) the mass density of water vapour as a fraction of the mass density of dry air
(c) the pressure exerted by water vapour as a fraction of the pressure exerted by air

(6) The dynamical tropopause coincides with the
(a) 2 PVU potential vorticity isopleth
(b) as the lowest level at which the temperature lapse rate decreases to 2 K/km or less, provided that the average lapse rate (eq. 1.18) between this level and all higher levels within 2 km does not exceed 2 K/km
(c) 2 PVU- and -2 PVU-potential vorticity isopleths

(7) The polar night winter lower stratospheric jet is
(a) stronger in the northern hemisphere than in the southern hemisphere
(b) stronger in the southern hemisphere than in the northern hemisphere
(c) is about equally strong in the southern hemisphere as in the northern hemisphere

(8) At 30°N latitude in the northern hemisphere the wind vector in an inertial oscillation
(a) rotates anticyclonically with time, with a period of 12 hours
(b) rotates cyclonically with time, with a period of 24 hours
(c) rotates anticyclonically with time with a period of 24 hours