Answer to problem 1

Section 1.14 of the lecture notes.

Answer to problem 2

The average vorticity within the circle with radius *r* around the centre of the cyclone is

$$\bar{\zeta}(r) = \frac{C(r)}{\pi r^2},$$

where $C(r)$ is the circulation around the circle with radius *r*.

For *r*≤*R* this implies that

$$\bar{\zeta}(r) = \frac{2\pi r}{\pi r^2}\frac{v_0 r}{R} = \frac{2v_0}{R} \equiv \zeta_0 = \text{constant}.$$

The vorticity inside the radius of maximaum wind (in the core of the cyclone) is constant.

For *r*>*R* this implies that

$$\bar{\zeta}(r) = \frac{2\pi r}{\pi r^2}\frac{v_0 R}{r} = \frac{2v_0 R}{r^2}.$$

This average vorticity is an area-weighted average of the constant vorticity in the core of the cyclone and the yet unknown vorticity-profile outside the core of the cyclone, i.e.

$$\bar{\zeta}(r) = \frac{2v_0 R}{r^2} = \frac{2\pi\left(\int_0^R \zeta_0 r dr + \int_R^r \zeta(r) r dr\right)}{\pi r^2}$$

or

$$v_0 R = \int_0^R \zeta_0 r dr + \int_R^r \zeta(r) r dr = \int_0^R \frac{2v_0}{R} r dr + \int_R^r \zeta(r) r dr = v_0 R + \int_R^r \zeta(r) r dr.$$

In other words,

$$\int_R^r \zeta(r) r dr = 0$$

for all values of *r*>*R*, so that

$$\boxed{\zeta(r) = 0}$$

<u>for all values of *r*>*R*, and</u>

$$\boxed{\zeta(r) = \frac{2v_0}{R}}$$

<u>for all values of *r*≤*R*.</u>

Answer to problem 3

Section 1.34 of the lecture notes.

Answer to problem 4

Section 1.35 of the lecture notes.

Answers to problem 5 (multiple choice)

(i) c
(ii) b
(iii) c
(iv) b
(v) a
(vi) a