Exam: Dynamical Meteorology-1
Date: November 11, 2016, 09:00-12:00

In this exam all symbols have their normal definitions.
Answers may be given in either English or Dutch

Problem 1 (2 points)
A hypothetical homogeneous hydrostatic atmosphere

A homogeneous atmosphere, is defined as an atmosphere in which density is constant, $\rho=\rho_0$. Assume that the homogeneous atmosphere is in hydrostatic balance. The pressure at the surface of the earth is $p=p_s=1000$ hPa. The temperature at the earth's surface is $T=T_s=15°C$.

(a) At which height is $p=0$?
(b) What is the temperature lapse rate?
(c) Is a homogeneous atmosphere with constant density observed. Why?

The vertical component of the equation of motion is

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \ .$$

with $g=9.81$ m s^{-2}. Assume that the atmosphere is an ideal gas, i.e.

$$p = \rho RT \ ,$$

with $R=287$ J K^{-1} kg^{-1}.

Problem 2 (2.5 points)
Tephigram

Figure 1 (below) displays a tephigram, showing observations made by a radiosonde, which was launched from Dallas (Texas) at 00:00 UTC on 19 April 2000. The surface dewpoint temperature is 21°C. The surface temperature is 29°C.

(a) Estimate the Lifting Condensation Level (LCL) (in hPa) of an air parcel at the surface from the tephigram.
(b) What temperature will the air parcel have at the LCL (approximately)?
(c) The temperature at the surface will rise during the day due to absorption of solar radiation by the surface of the earth. At what approximate surface temperature will convective clouds *start* to form spontaneously?
(d) At what height (in hPa) will the LCL be in that case ?
(e) What will be the approximate cloud top height (in hPa) if thunderstorms arrise spontaneously.

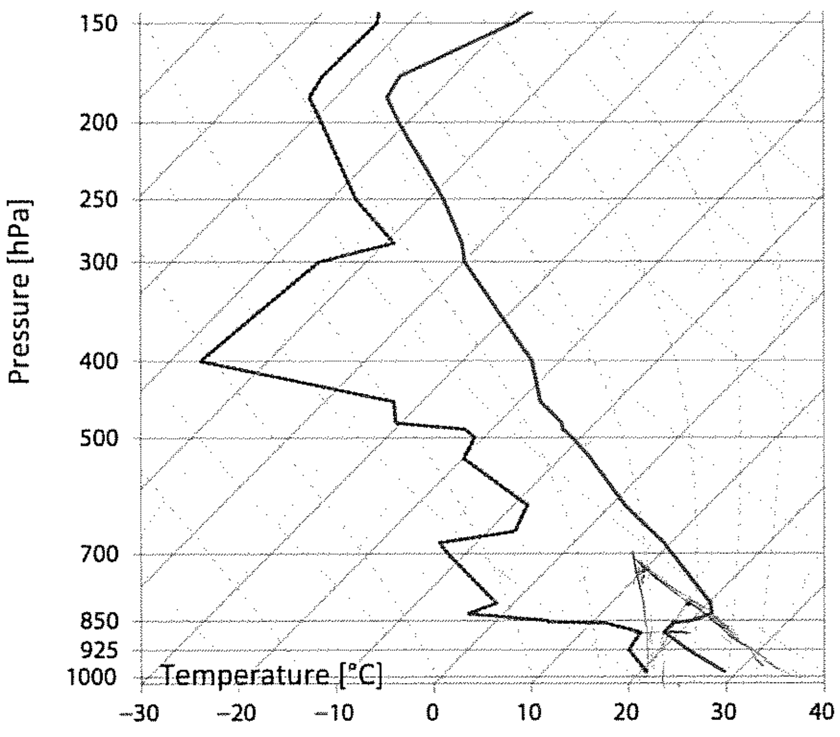

FIGURE 1. See problem 2.

Problem 3 (2 points)
Inertial motion

An air parcel is moving in the horizontal direction with a speed of 10 m s^{-1} under the influence of the Coriolis force. There is no horizontal pressure gradient and friction can be neglected. The simplified equations of horizontal motion which apply to the motion of this air parcel are

$$\frac{du}{dt} = fv \text{ and } \frac{dv}{dt} = -fu .$$

(a) Show that the trajectory of the air parcel is a circle (called an "inertial circle").
(b) What is the radius of this circle if f=10^{-4} s^{-1}?

For simplicity, assume that the Coriolis parameter, f, is constant.

Problem 4 (1.5 points)
Vertical distribution of water vapour and precipitable water

A good approximation to the vertical dependence of the density of water vapour is

$$\rho_v = \rho_{v,g} \exp\left\{-\frac{z}{H_v}\right\},$$

where $\rho_{v,g}$ represents the density of water vapour at the ground (z=0).
(a) Explain why this approximation is supported by the observations, which are plotted in **figure 2**.
(b) What would be your best estimate of the value of H_v?

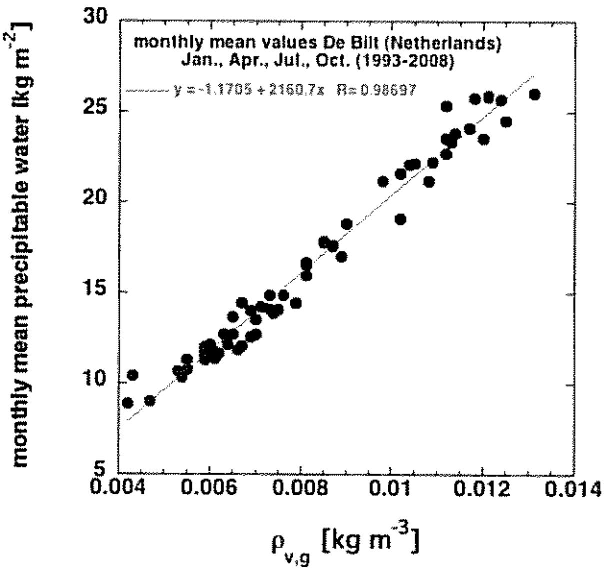

FIGURE 2. Monthly mean precipitable water in kg m^{-2} as a function of the monthly mean mass density of water vapour at the Earth's surface according to radiosonde measurements made at De Bilt in the years 1993 to 2008. The red line represents the best linear fit to the observations. See **problem 4**.

Problem ~~4~~ 5 (2 points)
Multiple choice
Indicate the "best" answer

(1) The three components of the momentum budget of the atmosphere relative to the earth can be written as

$$\frac{du}{dt} - \frac{uv\tan\phi}{a} + \frac{uw}{a} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + 2\Omega v\sin\phi - 2\Omega w\cos\phi, \quad (1)$$

$$\frac{dv}{dt} + \frac{u^2\tan\phi}{a} + \frac{vw}{a} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - 2\Omega u\sin\phi, \quad (2)$$

$$\frac{dw}{dt} - \frac{u^2+v^2}{a} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + 2\Omega u\cos\phi. \quad (3)$$

Because

$$\frac{u^2+v^2}{a} << g$$

we may approximate eq. (3) by

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + 2\Omega u\cos\phi \qquad (4)$$

The resulting approximate system of equations (1), (2) and (4)

(a) is physically consistent
(b) is physically consistent only for an adiabatic atmosphere
(c) is physically inconsistent

(2) The equation

$$\frac{d\Pi}{dt} = -\frac{R\Pi}{c_v}\vec{\nabla}\cdot\vec{v} + \frac{RJ}{c_v\theta},$$

describing the time rate of change of the Exner function (a measure of the pressure) of an air parcel, is derived from

(a) the law of mass conservation
(b) the law of mass conservation and the ideal gas law
(c) the law of mass conservation, the ideal gas law and the first law of thermodynamics

(3) The dewpoint temperature of an air parcel, which ascends adiabatically,

(a) decreases with increasing height
(b) increases with increasing height
(c) remains constant

(4) Ideally, the seabreeze, u, at the coast (u is the landward wind component perpendicular to the coast) is described by

$$\frac{\partial^2 u}{\partial t^2} + f^2 u = \frac{A\Omega}{\rho}\sin\Omega t,$$

where A is a constant. If A>0, this equation describes

(a) a forced damped inertial oscillation, consisting of two periods, $T_1=2\pi/f$ and $T_2=2\pi/\Omega$.
(b) a damped inertial oscillation with a period equal to $2\pi/f$.
(c) a forced oscillation, consisting of two periods, $T_1=2\pi/f$ and $T_2=2\pi/\Omega$.

(5) Teten's formula is an integrated form of the Clausius-clapeyron equation,

(a) which takes into account the dependence of the latent heat of condensation on pressure
(b) which takes into account the dependence of the latent heat of condensation on temperature
(c) which takes into account the dependence of the specific gas constant for water water vapour on temperature.