Examination **ICE AND CLIMATE (28 January 2016)**

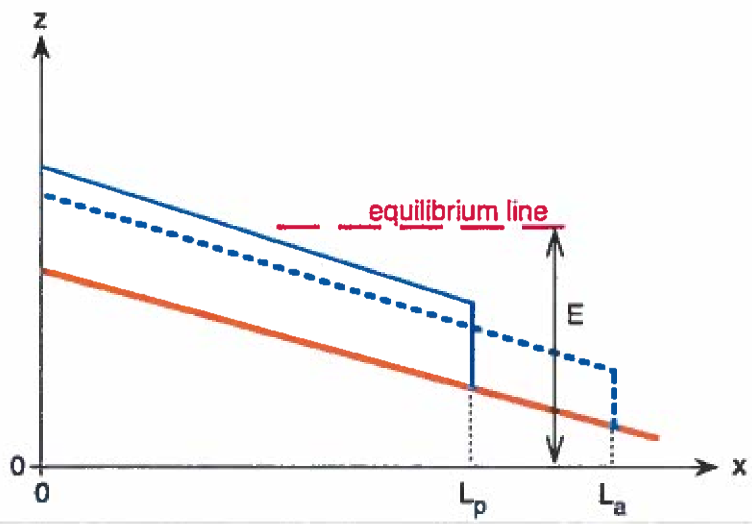
1. We study a cross section (x-axis) of an ice sheet on a bounded domain. The edges of the ice sheet are at $x = 0$ and $x = L$. The undisturbed bed (no ice load) is horizontal and at sea level. The balance rate increases linearly with height h: $\dot{b} = \beta(h - E)$, where E is the equilibrium-line altitude. Suppose that the ice behaves perfectly plastic (slope times ice thickness $= \tau_0 /(\rho_i g) = C$).
The ice sheet is in equilibrium with the climate and the solid earth (local isostacy).
a. What is the maximum ice thickness if $L = 1000$ km and $C = 12$ m? Ice and mantle density are 900 and 3500 kg m^{-3}, respectively.
b. When E is larger than a critical altitude E^*, the ice sheet cannot survive. Calculate E^* for the given parameter values.

2. Glaciers in the Alps are temperate: the ice temperature is at the freezing point everywhere in spite of the fact that the annual mean air temperature is well below 0°C.
a. How is this possible ?

In temperate glaciers sliding makes an important contribution to the total ice motion.
b. Explain the mechanism(s) that make sliding over a bumpy bed possible.

In the (sub)polar regions cold ice sheets are the rule.
c. Which processes determine the vertical temperature distribution close to the dome of a polar ice sheet ? (qualitative)

3.
Some glaciers surge at more or less regular intervals. During a surge ice velocities increase strongly for a short period of time (~ 1 year), and the glacier front advances rapidly. The net effect is that a glacier gets thinner and longer. During a surge increased sliding velocity implies more frictional heating and additional production of meltwater. The total frictional heating can be estimated from the loss of potential energy due to the surge.
To study this effect we consider a simple glacier of constant thickness H_p (before the surge) and constant (unit) width resting on a bed with constant slope s (see figure below). The bed is given by $b(x) = b_0 - sx$. The glacier length before the surge is L_p, after the surge $L_a = \lambda L_p$.



a. Find an expression for the total frictional heating (dissipation) D during the surge. Denote ice density (constant) by ρ and the acceleration of gravity by g. Assume that ice volume is conserved. *[Suggestion: compare the heights of the centres of gravity before and after the surge]*

b. Assume that the balance rate is a linear function of the surface height h [$\dot{b} = \beta(h - E)$].
Find the (pre-surge) equilibrium glacier length L_p for $b_0 = 3000\,\text{m}, E = 2400\,\text{m}, H_p = 100\,\text{m}, s = 0.1$.

After the surge the glacier surface has a lower mean elevation and is longer.
This implies a negative perturbation to the total mass budget.
c. Calculate the perturbation in the net balance rate for $\lambda = 1.1$, $\beta = 0.01\ \text{yr}^{-1}$

4. Much of our knowledge about past climates on geological time scales comes from the analysis of deep-sea sediments. The most widely used proxy is the $\delta^{18}O$ signal in the calcite shells of benthic forams, which is mainly determined by deep-sea temperature (T) and ice volume (V) according to

$\delta^{18}O = a + bT + cV \quad (a > 0;\ b < 0; c > 0)$

a. Explain briefly the mechanisms behind this relation.

We consider only Antarctic ice. A possible relation between Antarctic ice volume and deep-sea temperature is (T, T_0 and θ in °C; V_0 is a reference ice volume):

$V(T) = V_0(T_0 - T)\,e^{(T-T_0)/\theta} \quad (T \le T_0)$

The present-day deep-sea temperature is about 2°C.
b. Make a schematic graph of $V(T)/V_0$. The ice volume has a maximum. Is this realistic? Why?

c. Determine the parameters T_0 and θ by assuming that (i) there is no ice for T larger than 15°C, and (ii) the present ice volume is close to its maximum value (irrespective of changes related to sea-level fluctuations).

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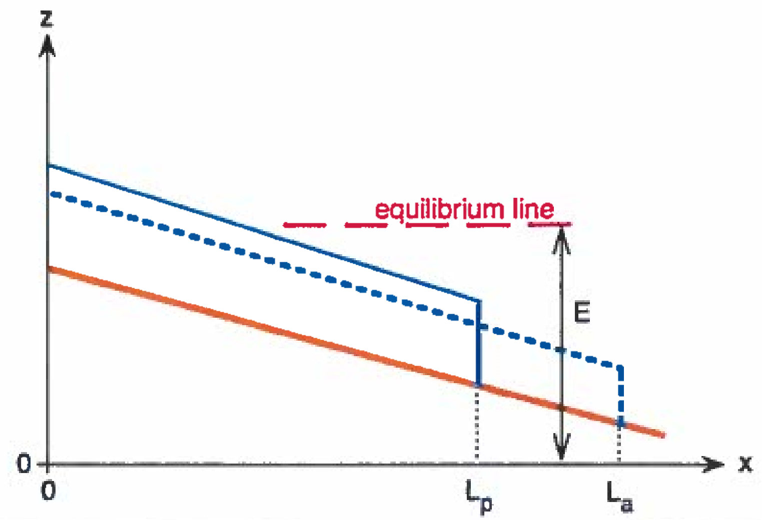

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