Ice and climate examination

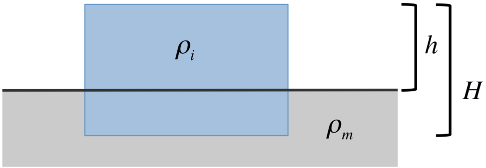
General remarks

- Allowed tools: calculator (not programmable)
- Tip: Read once through entire examination to decide with which question to start
- Tip: Read questions carefully → answer should only cover points that are specifically asked (→ text based question)

Hydrostatic pressure: $p = \rho\, g\, h$

$$\rho_i\, H\, g = (H - h)\, \rho_m\, g \tag{1}$$

$$h = \underbrace{\left(\frac{\rho_m - \rho_i}{\rho_m}\right)}_{\alpha} H \tag{2}$$



1.a

$$\frac{dh}{dx} H = C \quad (3)$$

$$\frac{dH\,\alpha}{dx} H = C \quad (4)$$

$$\int H\,dH = \frac{C}{\alpha} \int dx \quad (5)$$

$$H(x) = \sqrt{\frac{2\,C}{\alpha} x + K} \quad (6)$$

$$(7)$$

Boundary condition: $H(x = 0) = 0$

$$H(x) = \sqrt{\frac{2\,C}{\alpha} x} \quad (8)$$

$$H_{max} = H(x = L/2) = \sqrt{\frac{C}{\alpha} L} \approx 4019\,m \quad (9)$$

$$B_s = \int_0^L \dot{b}(x)\, dx = \beta \int_0^L (h - E)\, dx \geqslant 0 \tag{1}$$

$$B_s = \beta \int_0^L h(x)\, dx - \beta\, E^*\, L \stackrel{!}{=} 0 \tag{2}$$

$$E^* = \frac{1}{L} \int_0^L h(x)\, dx \tag{3}$$

Because the ice sheet is symmetric (around $x = L/2$), it is sufficient to perform the integration from $x = 0$ to $x = L/2$:

$$h(x) = \sqrt{2\, C\, \alpha\, x} \tag{4}$$

$$E^* = \frac{2}{L} \int_0^{L/2} h(x)\, dx = \frac{4}{3\, L} \sqrt{2\, C\, \alpha}\, x^{3/2} \Big|_0^{L/2} \tag{5}$$

$$E^* = \frac{2}{3} \sqrt{C\, \alpha\, L} \approx 1990\, m \tag{6}$$

Refreezing of percolating meltwater
In spring and summer, liquid water from surface melt percolates into the subjacent snowpack and refreezes. This transport of latent heat increases the local temperature to the melting point.

2.b

Ice flow around obstacles is facilitated by:

- **Regelation:** On the upstream side of the obstacle, the pressure is higher (→ melting). On the downstream side, the pressure is lower (→refreezing). This results in a flow of heat through the obstacle and a flow of meltwater around it.
- **Enhanced creep:** Due to the higher normal pressure on the obstacle, the ice softens (larger effective stress) and it is easier for the ice to move around the obstacle.

2.c

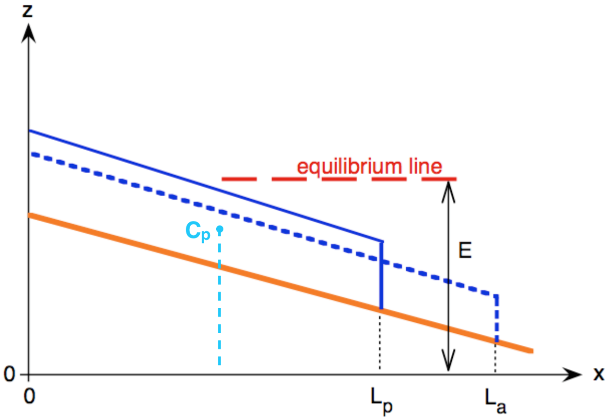
Close to the dome, horizontal temperature gradients are small, so only vertical processes are relevant. The most important ones are:

- Molecular conduction, downward advection of cold ice from the surface:

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(\alpha(z)\frac{\partial T}{\partial z}\right) - w\,\frac{\partial T}{\partial z} \tag{1}$$

- Supply of geothermal heat at the base of the ice sheet (→ lower boundary condition)

3.



The height of the centre of mass for the 'block glacier' before the surge is given by:

$$C_p = b_0 - \frac{s\,L_p}{2} + \frac{H_p}{2} \tag{1}$$

The height of the centre of gravity after the surge is (by using mass conservation $H_p\,L_p = H_a\,L_a$):

$$C_a = b_0 - \frac{s\,L_a}{2} + \frac{H_a}{2} = b_0 - \frac{s\,\lambda\,L_p}{2} + \frac{H_p}{2\,\lambda} \tag{2}$$

3.a

The difference in the centre of mass is hence:

$$C_a - C_p = -\frac{s\,\lambda\,L_p}{2} + \frac{H_p}{2\,\lambda} + \frac{s\,L_p}{2} - \frac{H_p}{2} \tag{3}$$

$$= \frac{s\,L_p}{2}(1-\lambda) + \frac{H_p}{2}(\frac{1}{\lambda} - 1) \tag{4}$$

The loss of potential energy (→ total dissipation) due to the surge is thus

$$\Delta P = g\,M\,(C_a - C_p) \tag{5}$$

where $M = \rho\,H_p\,L_p$ is the total mass.

The equilibrium length of the glacier is found by equating the total mass budget to zero:

$$\int_0^{L_p} \dot{b}(x)\, dx \stackrel{!}{=} 0 \qquad (1)$$

$$\beta \int_0^{L_p} (b_0 - s\, x + H_p - E)\, dx \stackrel{!}{=} 0 \qquad (2)$$

$$\beta \left[(H_p + b_0 - E)\, L_p - \frac{1}{2}\, s\, L_p^2 \right] \stackrel{!}{=} 0 \qquad (3)$$

$$L_p = \frac{2\,(H_p + b_0 - E)}{s} \qquad (4)$$

$$L_p = 14000\, m \qquad (5)$$

The mass budget after the surge is:

$$B_a = \int_0^{L_a} \dot{b}\, dx = \beta \left[(H_a + b_0 - E)\, L_a - \frac{1}{2}\, s\, L_a^2 \right] \qquad (1)$$

Using $L_a = \lambda\, L_p$ and $H_p\, L_p = H_a\, L_a$, this can be rewritten as:

$$B_a = \beta \left[(\frac{H_p}{\lambda} + b_0 - E)\, \lambda\, L_p - \frac{1}{2}\, s\, \lambda^2\, L_p^2 \right] \qquad (2)$$

The perturbation in the net balance rate is then:

$$\Delta \dot{b}_n = \frac{B_a}{L_a} = \beta \left[(\frac{H_p}{\lambda} + b_0 - E) - \frac{1}{2}\, s\, \lambda\, L_p \right] \approx -0.79\, m\, a^{-1} \qquad (3)$$

- **The sea water temperature part:** During the formation of the calcite shell of deep sea Foraminifera, variations in $\delta^{18}O$ are caused by temperature-dependent diffusion through the membrane of the microorganism.
- **The ice volume part:** During evaporation from the ocean surface, fractionation of oxygen isotopes takes place. Lighter water (with less ^{18}O) evaporates easier, so the ocean is enriched with ^{18}O when land ice forms.

4.c

$$\frac{V(T)}{V_0} = (T_0 - T)\exp\left(\frac{T - T_0}{\theta}\right) \tag{1}$$

Determine T_0 with (i):

$$\frac{V(T = 15^\circ C)}{V_0} \stackrel{!}{=} 0 \tag{2}$$

$$T_0 = 15^\circ C \tag{3}$$

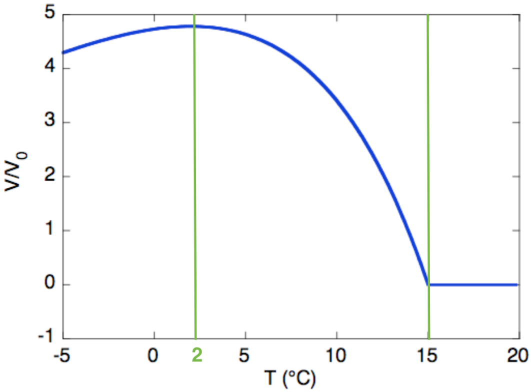
Determine θ with (ii):

$$(V/V_0)_{max} = \underbrace{\frac{T_0 - T}{\theta}}_{\stackrel{!}{=}1} \exp\left(\frac{T - T_0}{\theta}\right) - \exp\left(\frac{T - T_0}{\theta}\right) \stackrel{!}{=} 0 \tag{4}$$

$$\theta = (T_0 - T) = 13^\circ C \tag{5}$$

4.b

$$\frac{V(T)}{V_0} = (15^\circ C - T)\exp\left(\frac{T - 15^\circ C}{13^\circ C}\right) \quad (1)$$



4.b

Explanation of a maximum in ice volume:

- Low temperatures → low moisture-holding capacity of atmosphere → low snowfall rates
- High temperatures → high melt rates

So it is understandable that there is an optimum temperature for which the ice volume reaches a maximum.