EXAM OCEAN WAVES

3 November 2015, 9.30-11.30 hours

Three problems; all items have equal weight

Remark 1: answers may be written in English or Dutch

Remark 2: in all questions you may use $g = 10$ ms^{-2}, $\rho = 10^3$ kgm^{-3}, $\tau = 0.1$ Nm^{-1} and $\Omega = 7.3 \cdot 10^{-5}$ s^{-1}.

Problem 1

Consider a sea which for $t < 0$ has a perfectly smooth surface. At $t = 0$ a wind starts to blow with that has a velocity profile

$$U_a = \frac{u_*}{k} \ln\left(1 + \frac{z}{z_0}\right) .$$

After a few minutes the sea surface variations are dominated by two waves, with wavenumbers κ_1 and κ_2. These waves obey the dispersion relation

$$\sigma^2 = g\kappa + \frac{\tau}{\rho}\kappa^3 .$$

a. Derive expressions for the critical level of the two waves that contain the wavenumber and other model parameters.

b. Name and describe the physical mechanism that, after the wind has blown for a few minutes, causes the growth of waves.
Make a clear sketch to illustrate your arguments.
Limit your answer to 1 A4 page maximum.

After the wind has blown for an hour, a fully random wave field has evolved. This wave field has a frequency-direction spectrum $F(\sigma, \theta)$ that for $\sigma > \sigma_p$ (with σ_p the peak frequency) is characterised by

$$F(\sigma, \theta) = \beta\, \sigma^{-5}\, I(\theta) , \qquad I(\theta) = \begin{cases} \cos^2(\theta - \theta_w) & |\theta - \theta_w| < \frac{\pi}{2} \\ 0 & \text{for other values of } \theta , \end{cases}$$

where θ_w is the direction of the wind.

c. What is the physical unit of parameter β in the expression for $F(\sigma, \theta)$?
Show how you arrive at your answer.

d. Derive an expression for the energy density of the high frequency waves which are described by the given spectrum $F(\sigma, \theta)$.

Problem 2

The figure below shows the curve of the astronomical tide (sea level as a function of time, without effects of wind) for the station Do Son (Vietnam) for a period of 14 days. The grey blocks in the figure denote night time.

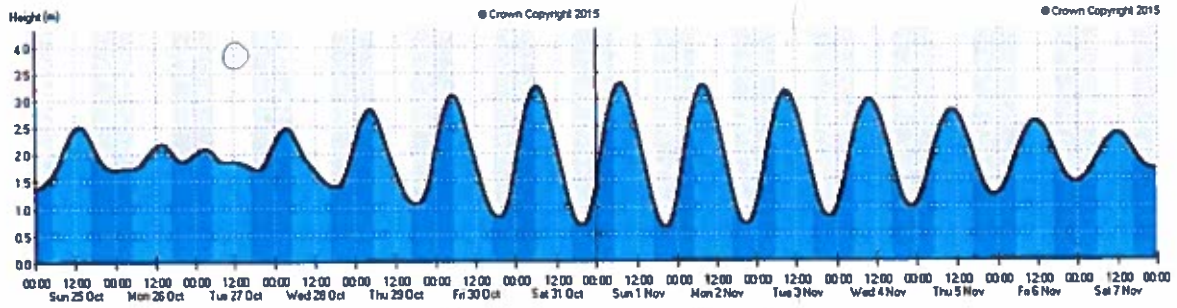

Figure 1: Tidal curve in Do Son (Vietnam)

a. Estimate the period of the dominant tidal constituent in Do Son.
   Also, explain qualitatively how this tidal constituent is generated.
   Provide your answer with a clear figure.
   Limit your answer to max. 1 A4 page.

b. On 27 October 2015 it was full moon, whilst the tide in Do Son was weak that day and had a period that differed from the typical period at all later days.
   Provide an explanation for these facts.
   Answer: maximum 0.5 A4 page.

c. Suppose that a curve of the astronomical tide as in the figure above would be given for the period 8-21 November. Would you expect in that record a different maximum tidal range than the maximum tidal range in the record shown in the figure above?
   Explain your answer and include a figure.
   Answer: maximum 0.5 A4 page.

Problem 3

The Atlantic Ocean can, to a first approximation, be schematised as a rectangular, semi-enclosed basin with a width B=5000 km and depth H=2500 m. Assume that the Coriolis parameter $f = 1 \times 10^{-4}$ s^{-1} and that the water motion is described by the frictionless linear depth-averaged shallow water equations. Consider the tidal motion in this basin that is forced at the open boundary with the southern ocean.

a. Compute the wavelengths of the free progressive waves in the basin that are forced by the semi-diurnal lunar tide (period 12 h 25 m).

b. Name all wave solutions forced by the semi-diurnal lunar tide in the Atlantic Ocean of which the amplitude shows exponential behaviour in one of the horizontal directions. Also, indicate at what location(s) these waves occur and compute their typical length scale.

END