Instituut voor Theoretische Fysica, Universiteit Utrecht

# RETAKE EXAM QUANTUM FIELD THEORY

13 March, 2014

- The duration of the exam is 3 hours.
- The exam is closed-book.
- Usage of a calculator and a dictionary is allowed.
- Use different sheets for each exercise.
- Write clearly, unclear writing will not be evaluated.
- Write your name and initials on every sheet handed in.
- Divide your available time wisely over the exercises.

## Problem 1 (Noether theorem) (20 points)

The Lagrangian for the nonlinear sigma model is

$$\mathscr{L} = -\frac{1}{2}\frac{\partial_\mu \phi^i \partial^\mu \phi^i}{(1+g\phi^2)^2}$$

Here $\phi^i$ with $i = 1, 2, 3$ is a three-component pseudo-vector, $\phi^2 \equiv \phi^i\phi^i$, where summation over $i$ is assumed. Also $g$ is a numerical coupling constant here.

1. Show that this Lagrangian is invariant under the transformation

$$\delta\phi = -\Lambda \times \phi + (1 - g\phi^2)\xi + 2g(\xi \cdot \phi)\phi\,,$$

where $\Lambda$ and $\xi$ are two independent three-component vectors (also independent on space-time coordinates).

2. Calculate the corresponding Noether currents and show that they are conserved. Distinguish three vector and three axial-vector currents associated with $\Lambda$ and $\xi$.

*Hint.* Use the general formula for a conserved Noether current

$$J^k_n = -\frac{\partial \mathscr{L}}{\partial(\partial_k \phi_I)}\Phi_{I,n}\,.$$

where $\delta\phi_I(x) = \sum_{1\leq n\leq s} \Phi_{I,n}\delta\omega_n$. Here all $\delta\omega_n$, $n = 1, \ldots, s$ are independent infinitesimal transformation parameters, also independent of the space-time coordinates $x$.

## Problem 2 (15 points)

Show that

$$\mathrm{Tr}(\not{p}_1\gamma^\mu\not{p}_2\gamma_\mu) = a(p_1 \cdot p_2)$$

and compute the constant $a$. Here $\not{p}$ is the standard notation for $\not{p} \equiv \gamma^\mu p_\mu$.

## Problem 3 (15 points)

Consider the generating functional of the free Klein-Gordon theory

$$Z[J] = Z_0[J] \exp\Big[ -\frac{1}{2}\int \mathrm{d}^4x\mathrm{d}^4y J(x)\Delta(x-y)J(y)\Big] ,$$

where $\Delta$ is the Klein-Gordon propagator.

Evaluate the four-point function

$$\langle 0|T(\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4))|0\rangle = \frac{1}{Z[0]}\frac{1}{i^4}\frac{\delta}{\delta J(x_1)}\frac{\delta}{\delta J(x_2)}\frac{\delta}{\delta J(x_3)}\frac{\delta}{\delta J(x_4)}Z[J]|_{J=0}$$

and represent the result pictorially.

## Problem 4 (25 points)

Consider a real scalar field $\phi$ with the Lagrangian

$$\mathscr{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \lambda\phi^3 - g\phi^4$$

1. Write down Feynman rules for this theory in coordinate space.
2. Draw the one-loop and two-loop diagrams which contribute to the self-energy for $\phi$.
3. Write Green's function in momentum space in terms of the self-energy $\Sigma(p)$.

## Problem 5 (25 points)

Consider the Lagrangian density for a massive vector field (work in units $\hbar = c = 1$)

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu .$$

1. Find the Hamiltonian of the model and the corresponding Hamiltonian equations of motion.
2. What is a number of physical degrees of freedom carried by the field $A_\mu$?