DEPARTEMENT NATUUR- EN STERRENKUNDE
UNIVERSITEIT UTRECHT

RETAKE EXAM Quantum Field Theory - NS-TP401M

Thursday, March 12, 2015, 13:30-16:30, BBG083.

1) Start every exercise on a separate sheet. Write on each sheet: your name and initials, and your studentnumber.
2) Please write legibly and clear. Unreadable handwriting cannot be marked!
3) The exam consists of **three** exercises and replaces the total final mark.
4) No lectures notes or any other material (books, calculators, ...) are allowed. A formularium instead is given.

Formularium

We use natural units, in which $c = \hbar = 1$ in this exam. The Minkowski metric in four spacetime dimensions is $\eta_{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1)$. The Dirac matrices $(\gamma^\mu)_\alpha{}^\beta$ satisfy

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \ . \tag{1}$$

The Delta function has an integral representation given by

$$\int_{-\infty}^{+\infty} \mathrm{d}p\, e^{ipx} = 2\pi\delta(x) \ . \tag{2}$$

1. Short questions (3 points)

Answer the following questions (be brief when you can):

i) What is a Dirac spinor ? Write down the Dirac equation and explain all symbols.

ii) What is the $i\epsilon$-prescription for propagators in quantum field theory, and why is it needed ? What does it have to do with the Wick rotation ?

iii) What is "renormalization" in quantum field theory and what does the concept "renormalizibility of a theory" mean ?

2. Massive vector fields (4 points)

Consider the following Lagrangian for a massive vector field coupled to a Dirac spinor in four spacetime dimensions,

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu V_\nu - \partial_\mu V_\nu)^2 - \frac{1}{2}M^2 V_\mu V^\mu + ieV_\mu \bar{\psi}\gamma^\mu \psi - \bar{\psi}(\gamma^\mu \partial_\mu + m)\psi \ . \tag{3}$$

i) Write down the equations of motion for all the fields.

ii) Show that the propagator (in momentum space) for the massive vector field V_μ is proportional to the expression

$$\frac{\eta_{\mu\nu} + p_\mu p_\nu / M^2}{p^2 + M^2} \ . \tag{4}$$

iii) Determine the (mass) dimensions of the fields and coupling constants for $M = 0$. Is the theory renormalizable by power counting ?

iv) Now look at $M \neq 0$ and focus on the longitudinal mode of the vector field

$$V_\mu = \frac{1}{M}\partial_\mu \phi \ . \tag{5}$$

Rewrite the Lagrangian for this longitudinal mode, determine again the mass dimensions of the fields and coupling constants and find out whether the theory is renormalizable by power counting.

3. Renormalizability of Yukawa couplings ? (3 points)

Consider a field theory for a massive Dirac spinor ψ and a scalar field ϕ in $d = 2$ spacetime dimensions. The free Lagrangian (density) reads

$$\mathcal{L}_0 = -\bar{\psi}(\gamma^\mu \partial_\mu + M)\psi - \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2 \phi^2 \ . \tag{6}$$

We consider two types of interactions between these fields,

$$\mathcal{L}_1 = g_1 \, (\bar{\psi}\psi) \, \phi \ , \qquad \mathcal{L}_2 = g_2 \, (\bar{\psi}\gamma^\mu \psi) \, \partial_\mu \phi \ . \tag{7}$$

i) Write down the Feynman rules (in momentum space) for the two theories given by $\mathcal{L}_0 + \mathcal{L}_1$ and $\mathcal{L}_0 + \mathcal{L}_2$.

ii) Consider the one-loop diagrams contributing to the fermion self-energy and give the explicit expressions for the two theories. Are the diagrams divergent and, if so, indicate the counterterms that one needs to absorb all the (one-loop) divergencies.

iii) What are the mass dimensions of the fields and the coupling constants in the two theories ? Are these theories renormalizable by power counting and why (not) ?