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In 2005/2006, the course NS-TP526M was given by dr. K. Peeters.

String Theory (NS-TP526M)
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Question 1

Classical closed bosonic string propagates in 5-dimensional Minkowski space-time according to

$$\begin{aligned} X^0 &= \kappa\tau, \\ X^1 &= a\sin n\sigma\cos n\tau, \\ X^2 &= a\sin n\sigma\sin n\tau, \\ X^3 &= b\sin m\sigma\cos m\tau, \\ X^4 &= b\sin m\sigma\sin m\tau. \end{aligned}$$

Here n, m are integers.

a) Show that the Virasoro constraints are satisfied provided the parameters of the solution are related as

$$\kappa^2 = a^2n^2 + b^2m^2$$

b) Compute the energy of the string and the angular momenta $J_1 \equiv J_{12}$ and $J_2 \equiv J_{12}$ corresponding to rotation of string in spatial planes 12 and 34 respectively.

c) Show that the energy is related to the angular momenta as

$$E = \sqrt{\frac{2}{\alpha'}(nJ_1 + mJ_2)}, \quad \text{where} \quad \alpha' = \frac{1}{2\pi T}.$$

Question 2

Consider classical closed string in the light-cone gauge. Show that if the level-matching condition is not satisfied then the Lorentz generators J^{i-} are not conserved quantities (in time) anymore.

Question 3

What is a conformal operator with conformal dimension Δ (give a definition)?

Question 4

Consider closed fermionic string. Find the propagator for fermions in the NS sector ($\tau > \tau'$):

$$\langle\psi_+^\mu(\tau,\sigma), \psi_+^\nu(\tau',\sigma')\rangle = T\left(\psi_+^\mu(\tau,\sigma), \psi_+^\nu(\tau',\sigma')\right) - :\psi_+^\mu(\tau,\sigma), \psi_+^\nu(\tau',\sigma'):,$$

where T stands for the operation of time ordering.

Question 5

How many (real) components has a Majorana-Weyl spinor of 10-dimensional Minkowski space-time?

Question 6: Spiky strings! (bonus)

Consider classical bosonic string propagating according to

$$\begin{aligned} X^0 &= t = \tau, \\ \vec{X} &= \vec{X}(\sigma^+) + \vec{X}(\sigma^-). \end{aligned}$$

Here $\vec{X} = \{X^i\}, i = 1, \ldots d$ and

$$\begin{aligned} \vec{X}(\sigma^-) &= \frac{\sin(m\sigma^-)}{2m}\mathbf{e}_1 + \frac{\cos(m\sigma^-)}{2m}\mathbf{e}_2 \\ \vec{X}(\sigma^+) &= \frac{\sin(n\sigma^+)}{2n}\mathbf{e}_1 + \frac{\cos(n\sigma^+)}{2n}\mathbf{e}_2 \end{aligned}$$

where $\mathbf{e}_1$ and $\mathbf{e}_2$ are two unit orthagonal vectors and the ratio $\frac{n}{m}$ is an integer.

a) Show that this configuration satisfies the Virasoro constraints.

b) Show that there are points on the string where $\vec{X}' = 0$. Show that at these points $\dot{\vec{X}}^2 = 1$, i.e. these points move with the speed of light — these are *spikes*.

c) Let $m = 1$ and $n = k - 1$. Show that k is the number of spikes.