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In 2005/2006, the course NS-TP526M was given by dr. G. Arutyunov.

String Theory (NS-TP526M)
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Question 1

A classical open bosonic string propagates in 3-dimensional Minkowski space-time according to

$$\begin{aligned} X^0 &= L\tau \\ X^1 &= L\cos\sigma\cos\tau \\ X^2 &= L\cos\sigma\sin\tau \end{aligned}$$

a) Find the area of the world-sheet swept by the string in one period of rotation.

b) Compute the angular momentum $J \equiv J_{12}$ and the mass M^2 for this string motion. Verify that $J \equiv \alpha' M^2$.

Question 2

Explain what the level-matching constraint is.

Question 3

Consider a closed string in the light-cone gauge. Define the transversal Virasoro generators as follows:

$$\begin{aligned} L_m^\perp &= \frac{1}{2}\sum_{n=-\infty}^{\infty} \alpha_{m-n}^i \alpha_n^i, \\ \bar{L}_m^\perp &= \frac{1}{2}\sum_{n=-\infty}^{\infty} \bar{\alpha}_{m-n}^i \bar{\alpha}_n^i. \end{aligned}$$

a) Compute the action of the transverse Virasoro generators on string coordinates, i.e., find

$$\{L_m^\perp, X^i(\sigma,\tau)\} = ? \qquad \{\bar{L}_m^\perp, X^i(\sigma,\tau)\} = ?$$

b) What is the Poisson bracket $\{L_m^\perp, L_n^\perp\}$?

Question 4

A classical open string moves in three-Minkowski space. Assume that the motion (in the light-cone gauge) is defined by $x_0^- = x_0^i = 0$, and the vanishing of all transversal oscillators α_n^i except

$$\alpha_1^1 = \left(\alpha_{-1}^1\right)^* = a,$$

where a is a dimensionless real constant.

a) Construct explicitly the string coordinates $X^0(\sigma,\tau), X^1(\sigma,\tau)$ and $X^3(\sigma,\tau)$.

b) What further restrictions are needed to describe a string which oscillates in the (X^1, X^2) plane and has *zero* momentum in this plane?

c) For the last case compute the (time-dependent) length and the energy of the string in terms of a and the Regge slope parameter α'.

Question 5 (Bonus)

Solve the conformal Killing equations for the case of an open string. Describe the conformal Killing vectors which leave the midpoint $\sigma = \frac{\pi}{2}$ of the open string taken at $\tau = 0$ fixed.