Instituut voor Theoretische Fysica, Universiteit Utrecht

MIDDLE TERM EXAM STRING THEORY

Thursday, 17, 2008

- The duration of the test is 3 hours.
- *Only the lecture notes may be consulted during the test.*
- Use different sheets for each exercise.
- Write your name and initials on every sheet handed in.
- Divide your available time wisely over the exercises.

Problem 1

Consider a classical closed bosonic string which propagates in 5-dimensional Minkowski space-time according to

$$\begin{aligned} X^0 &= \kappa\tau\,, \\ X^1 &= A\cos n\sigma\cos n\tau\,, \\ X^2 &= A\sin n\sigma\cos n\tau\,, \\ X^3 &= B\sin m\sigma\cos m\tau\,, \\ X^4 &= B\sin m\sigma\sin m\tau\,. \end{aligned}$$

Here n, m are integers. Questions:

1. To satisfy the Virasoro constraints, the parameter κ of this solution must be related to the other parameters A, B and n, m. Find this relationship.

2. Compute the energy of the string.

3. How many non-trivial angular momentum components $J^{\mu\nu}$ are carried by the string? Find them.

4. Find the length of the string at the following moment of time: $\tau = \frac{\pi}{2n}$.

5. What kind of motion exhibits the string in the plane which passes through the coordinate axes X^1 and X^2?

Problem 2

Explain, by using reparametrization freedom of the closed string, that one can not fix the light-cone gauge by imposing the condition $X^+ = 0$?

Problem 3

Consider a point particle moving in four-dimensional Minkowski space and interacting with the electromagnetic field with the potential A_μ, $\mu = 0, 1, 2, 3$. The action is[1]

$$S = \frac{1}{2}\int_{\tau_1}^{\tau_2} \mathrm{d}\tau \Big(\frac{1}{e}\dot{x}^2 - em^2\Big) - q\int_{\tau_1}^{\tau_2} \mathrm{d}\tau\, A_\mu(x)\dot{x}^\mu\,.$$

Here e is an auxiliary field (one-dimensional metric) and a constant q is the electric charge. First, derive the equations of motion for x^μ. Second, impose the static gauge $x^0 = \tau$ and find the corresponding Hamiltonian.

Problem 4

Consider classical closed string in the light-cone gauge.

1. Explain the appearance of the level-matching condition $\mathcal{V} = 0$.
2. What is the value of the Poisson bracket $\{\mathcal{V}, J^{i-}\}$. To answer this question, you should recall the meaning of $\mathcal{V}$ as the generator of rigid σ-translations.

Problem 5

Consider the Virasoro constraints $T_{\alpha\beta} = 0$ for the closed string.

1. Substitute in these constraints the light-cone gauge choice and solve them for the unphysical fields X^- and P^-.
2. Explain how the light-cone Hamiltonian H is related to P^-.

[1] In unites where the speed of light c is taken to be $c = 1$.