Instituut voor Theoretische Fysica, Universiteit Utrecht

END EXAM STRING THEORY

Thursday, June 26, 2008

- Use different sheets for each exercise.
- Write your name and initials on every sheet handed in.
- The lecture notes "Lectures on String Theory" may be consulted during the test, as well as your own notes.
- Some exercises require calculations. Divide your available time wisely over the exercises.

Problem 1 (*Classical open bosonic strings*)

Consider the following parametric equations:

$$\begin{aligned} X^0 &= 3A\tau\,, \\ X^1 &= A\cos(3\tau)\cos(3\sigma)\,, \\ X^2 &= A\sin(\beta\tau)\cos(\gamma\sigma)\,, \end{aligned} \qquad (1)$$

where A is a constant and β and γ are arbitrary positive coefficients.

1. Fix β and γ so that the equations above describe an open string solution, fulfilling also the non-linear constraints $T_{\alpha\beta} = 0$ *(in all the remaining parts of this exercise, always assume these values of β and γ)*. Write down the explicit expression of the solution in the form:

$$X^\mu(\tau,\sigma) = X_L^\mu\,(\tau-\sigma) + X_R^\mu\,(\tau+\sigma)\,.$$

   Which boundary conditions does the solution fulfill in the various space-time directions?

2. For what values of the modes x^μ, p^μ and α_n^μ does the general open string solution reproduce the expressions (1)?

3. Compute the center-of-mass four-momentum P^μ and the angular momentum $J^{\mu\nu}$ for the solution under consideration, and show that they are conserved.

Problem 2 (*Counting Virasoro descendants*)

Let $|\Phi\rangle$ be a primary state which is an eigenstate of the number operator N with an eigenvalue N_Φ: $N|\Phi\rangle = N_\Phi|\Phi\rangle$. How many algebraic independent Virasoro descendants one has at a fixed level $N_\Phi + n$? Motivate your answer.

Problem 3 (*Graviton and dilaton states in covariant quantization*)

Examine the closed string states $\zeta_{\mu\nu}\alpha^\mu_{-1}\bar{\alpha}^\nu_{-1}|p\rangle$ with $\zeta_{\mu\nu} = \zeta_{\nu\mu}$.

1. Show that the Virasoro constraints imply the conditions $p^2 = 0$ and $p_\mu\zeta^{\mu\nu} = 0$.
2. Exhibit the null states that generate the physical state equivalence $\zeta^{\mu\nu} \sim \zeta^{\mu\nu} + p^\mu\epsilon^\nu + p^\nu\epsilon^\mu$, which holds for $p^2 = 0$ and $p_\mu\epsilon^\mu = 0$.
3. Show that there are $(d-2)(d-1)/2$ independent physical degrees of freedom in $\zeta_{\mu\nu}\alpha^\mu_{-1}\bar{\alpha}^\nu_{-1}|p\rangle$ for each value of p_μ which satisfies $p^2 = 0$. These are the degrees of freedom of a graviton and a scalar particle called dilaton.

Problem 4 (*Fermionic string*)

By using the equations of motion for fermionic string in the superconformal gauge, show the conservation of the fermionic current

$$G_\alpha = \frac{1}{4}\rho^\beta\rho_\alpha\psi^\mu\partial_\beta X_\mu\,.$$

Problem 5 (*Propagator for fermions*)

Consider closed fermionic string. Find the propagator for fermions in the NS sector ($\tau > \tau'$):

$$\langle\psi^\mu_+(\tau,\sigma),\psi^\nu_+(\tau',\sigma')\rangle = T\Big(\psi^\mu_+(\tau,\sigma)\psi^\nu_+(\tau',\sigma')\Big) - :\psi^\mu_+(\tau,\sigma)\psi^\nu_+(\tau',\sigma'):,$$

where T stands for the operation of time ordering.

Problem 6 (Bonus) *Spiky strings!*

Consider classical bosonic string propagating according to

$$X^0 = t = \tau \,,$$
$$\vec{X} = \vec{X}(\sigma^+) + \vec{X}(\sigma^-) \,.$$

Here $\vec{X} = \{X^i\}$, $i = 1, \ldots d$ and

$$\begin{aligned} \vec{X}(\sigma^-) &= \frac{\sin(m\sigma^-)}{2m}\mathbf{e}_1 + \frac{\cos(m\sigma^-)}{2m}\mathbf{e}_2, \\ \vec{X}(\sigma^+) &= \frac{\sin(n\sigma^+)}{2n}\mathbf{e}_1 + \frac{\cos(n\sigma^+)}{2n}\mathbf{e}_2 \,, \end{aligned}$$

where $\mathbf{e}_1$ and $\mathbf{e}_2$ are two unit orthogonal vectors and the ratio $\frac{n}{m}$ is an integer.

Questions:

- Show that this configuration satisfies the Virasoro constraints.
- Show that there are points on the string where $\vec{X}' = 0$. Show that at these points $\dot{\vec{X}}^2 = 1$, i.e. these points move with the speed of light – these are *spikes*.
- Let $m = 1$ and $n = k - 1$. Show that k is the number of spikes.