MIDTERM EXAM SUBATOMIC PHYSICS - Friday March 21, 2014

NS-369B

For this exam you are allowed to use your book (Subatomic Physics by Henley and Garcia), the notes from the first lecture about relativistic kinematics, and a simple calculator. Other materials are not allowed.

Important note: Start the solution of each exercise on a different sheet of paper. Explain your answers clearly, and don't forget to write units.

Grading: The exam consists of two questions, where the last one is a bit more involved than the first one. The weight of the two questions will roughly be 4/7 (i.e. one point per sub-question).

Notation: The symbol E will always stand for total relativistic energy, unless stated otherwise.

Exercise 1: Time Of Flight

A common way to identify particles is by measuring their time of flight (TOF). The idea behind a TOF measurement is that if the momentum of a particle can be determined very accurately (for example by measuring the curvature of its path in a magnetic field), then by measuring the time it takes for a particle to travel from the collision point (where it was created) to a detector at a distance L we can determine its mass[1].

A TOF detector has a time resolution Δt, which means that it can only distinguish particles if they arrive at the detector with a time difference greater than Δt. In this exercise we are going to find how the TOF detector's time resolution affects its mass resolution Δm, i.e., the minimum mass difference of two particles needed to distinguish them.

In the following, we will assume that the time resolution of the TOF detector is $\Delta t = 10^{-10}$s, and the distance between the collision point and the detector is $L = 1$m. Furthermore, we assume that all particles are created in one collision, i.e., they are created at the same time and at the same position.

a) Suppose there are two particles with different masses m_1 and m_2, but with the same momentum p. Show that the difference of arrival time of the two particles Δt is given by:

$$\Delta t \equiv t_2 - t_1 = \frac{L}{pc^2}(E_2 - E_1), \tag{0.0.1}$$

where $t_{1,2}$ are the arrival times of particles 1 and 2 respectively. Show also that in the non-relativistic limit ($\beta \ll 1, \gamma = 1 + O(\beta^2)$), this reduces to:

$$\Delta m \approx \frac{p}{L}\Delta t, \tag{0.0.2}$$

where $\Delta m \equiv m_2 - m_1$.

b) Particle identification using time of flight is most challenging when the masses of the different particles are nearly the same, i.e., $m_1 \approx m_2$. Assuming that particle 1 has a velocity $\beta_1 = 0.1$, show that particle 2 can be distinguished from particle 1 if its mass is at least 0.3% different from the mass of particle 2.

c) Now consider the relativistic case ($\gamma \gg 1, \beta \approx 1$). Show that the difference in arrival time in Eq. (0.0.1) is approximately:

$$\Delta t \approx \frac{Lc}{2p^2}(m_2^2 - m_1^2). \tag{0.0.3}$$

d) What is the highest momentum at which we can still distinguish a kaon and a pion? Give your answer in units of GeV/c. *Hint:* $m_\pi = 140 MeV/c^2$*, and* $m_K = 494 MeV/c^2$

Exercise 2: Two-Body Decay

A very common type of decay process in particle physics is the two-body decay, i.e., an unstable particle decaying into two other (stable or unstable) particles. Examples are $\pi^0 \to \gamma\gamma$, $\rho^0 \to \pi^+\pi^-$, etc.

Whether such a process occurs in nature and how likely it is depends on various things. First of all, there needs to be a force in nature which actually makes the transition possible, for example neutral pion decaying into two photons is an electromagnetic process, whereas the ρ^0 decaying into two pions is a strong process. Secondly, the process needs to be kinematically allowed, i.e., it needs to conserve energy and momentum[2]. In this question we're going to look into the kinematics of two-body decays.

a) Suppose there is an unstable particle of species A and mass m_A, decaying into two particles, one of species B and one of C, with masses m_B and m_C respectively. Show that for this decay to happen, we must have that $m_A^2 \geq m_B^2 + m_C^2$. *Hint: Start by realizing that* $m_A^2 c^4 = E_A^2 - \mathbf{p}_A^2 c^2$.

[1] In this exercise we make the simplification that every particle travels in a straight line, which is of course not the case if there is a magnetic field.

[2] One last important ingredient, which we will discuss in the second half of this course, is that the likeliness of a process depends on the amount of available phase space.

b) The result of part a) suggests that the process $\gamma \to e^+e^-$ (pair-creation) is not kinematically allowed. With similar logic we can see that also the process $e^+e^- \to \gamma$ is also not kinematically allowed. Explain why the process $e^+e^- \to \gamma \to e^+e^-$ in fact *does* happen. *Hint: the intermediate photon only exists for a brief time, so what can we say about its energy?*

c) Now consider the process $\pi^0 \to \gamma\gamma$ in the rest frame S of the pion. Due to momentum conservation, we know that the two photons will be created back-to-back. Write down the four-momenta of the pion and the two photons, in terms of the mass of the pion and θ_1 (see Fig. 1). *Hint: you should find that the energy of both photons is the same:* $E_{\gamma,1} = E_{\gamma,2} = m_\pi c^2/2$.

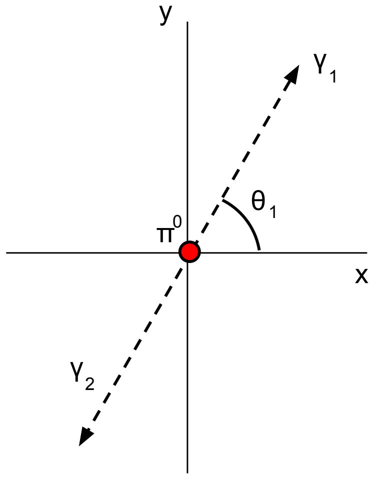

Figure 1: Decay of a neutral pion in the pion's rest-frame. Photon 1 is emitted under an angle θ_1 w.r.t. the x-axis.

In part c) you found that in the rest frame of the pion, the energy of both photons was the same, and uniquely determined by the mass of the pion. Hence, if we were to write down the energy probability-distribution of the photons in the rest frame, we would get something like:[3] $P(E_{\gamma,i}) = \delta(E_{\gamma,i} - m_\pi c^2/2)$, where $i \in \{1, 2\}$.

d) Write down the four-momenta of the pion and the two photons, but now in a frame S', boosted in the *negative* x-direction w.r.t. the pion rest frame S. What is the direction of motion of the pion in the boosted frame S'?

e) Express the mass of the pion in terms of the energies of the photons E_1 and E_2, and the opening angle α, where the opening angle is defined as the angle between the two photon trajectories.

From now on we are going to consider photon 1, i.e., we will assume $i = 1$, and no longer write the subscript. We wish to see how the probability-distribution of the energy of the photon is affected by the Lorentz boost, i.e., we want to calculate $P(E'_\gamma)$. It turns out that the easiest way to do this is to relate this energy probability-density to the probability-density of the cosine of the angle θ under which the photon is emitted, i.e.,

$$P(E'_\gamma)dE'_\gamma = P(\cos\theta)d\cos\theta \to P(E'_\gamma) = \frac{d\cos\theta}{dE'_\gamma}P(\cos\theta). \tag{0.0.4}$$

Knowing that in the rest frame of the pion $P(\cos\theta)$ has to be costant and normalized, we get:

$$1 = \int_{-1}^{1} P(\cos\theta)d\cos\theta \to P(\cos\theta) = \frac{1}{2}$$

.

f) Use this to show that:

$$P(E'_\gamma) = \begin{cases} 0 & \text{if} \quad E'_\gamma < a \\ \frac{1}{|\mathbf{p}'_\pi|c} & \text{if} \quad a < E'_\gamma < b \\ 0 & \text{if} \quad E'_\gamma > b, \end{cases} \tag{0.0.5}$$

where $a \equiv (E'_\pi - |\mathbf{p}'_\pi|c)/2$, and $b \equiv (E'_\pi + |\mathbf{p}'_\pi|c)/2$. Explain, for each of the three cases ($E'_\gamma < a, a < E'_\gamma < b, E'_\gamma > b$) why the probability has the value that is listed in equation 0.0.5.

Hint: first show that

$$E'_\gamma = \frac{\gamma m_\pi c^2}{2}(1 + \beta\cos\theta) \tag{0.0.6}$$

[3] In fact, since the π_0 has a finite lifetime, the actual energy-distribution would be more like a Breit-Wigner distribution (see for example Fig. (5.17)). This can however be well approximated by a δ-function in case of relatively long lifetimes.