Panos Christakoglou[*,†]

Subatomic physics

Mid-term exam

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[*]P.Christakoglou@uu.nl - Utrecht University, Leonard S. Ornsteinlaboratorium Princetonplein 1, 3584CC Utrecht, The Netherlands
[†]Panos.Christakoglou@nikhef.nl - Nikhef, Science Park 105, 1098XG Amsterdam, The Netherlands

Please remember to:

- Add your name and student ID on every sheet you plan to hand in
- Write as clear as possible
- Use the white exam sheets to hand in your answers for all the questions. This applies also to the multiple choice exercise!!!

Note that the points of each question is indicated in brackets at its end.

Exercise 1

1. If we can not find an observer for which the order of two events can be reverted then these events **(0.5)**

   a. their space-time 4-vector is space-like

   b. can be causally connected

   c. can only be both in the absolute past

   d. their space-time 4-vector is light-like

2. A spaceship flies away from its base on planet Earth, travelling with a constant velocity half the speed of light. The captain of the fleet stationed at the headquarters on Earth calls in to check on the crew after 1 month. The time elapsed for the crew of the spaceship is **(0.5)**

   a. is not altered since its motion can be described by Newtonian mechanics

   b. shortened by the Lorentz factor γ

   c. extended by the Lorentz factor γ

   d. contracted by the inverse of the Lorentz factor

3. A system is described by a Lagrangian of the form $\mathrm{L} = f(x, y, \dot{x}, \dot{y}, \dot{z})$, where x, y, z the standard Cartesian coordinates. Which of the following is correct? **(0.5)**

   a. The system is not invariant under any transformation

   b. The z–coordinate of momentum is a constant of motion

   c. The z–coordinate of angular momentum is conserved

   d. The azimuthal angle is a constant of motion

4. A group is called non-Abelian when **(0.5)**

   a. its elements do commute

   b. its elements do not commute

   c. its generators are non-Hermitian

   d. its generators are Hermitian

5. The Λ–particles containing a combination of *uds* quarks are **(0.5)**

   a. fermions like any baryon

   b. bosons like any baryon

   c. fermions like any meson

   d. bosons like any meson

6. Isospin symmetry is a characteristic of **(0.5)**

a. weak interactions
b. electromagnetic interactions
c. strong interactions
d. gravity

7. Strangeness conservation is violated in **(0.5)**

a. all interactions of the Standard Model
b. electromagnetic interactions
c. weak interactions
d. strong interactions

8. A minimum ionizing muon in iron has a momentum of the order of (see fig. 0.1) **(0.5)**

a. few GeV
b. few KeV
c. few eV
d. few MeV

9. Which technique is more suitable for identifying hadrons beyond a given momentum threshold? **(0.5)**

a. Transition radiation
b. Scintillation technique
c. Measurement of ionisation energy loss (dE/dx)
d. Cherenkov radiation

10. The radiation length **(0.5)**

a. is the average distance needed for the energy of an electron to be reduced by bremsstrahlung to half of its initial value
b. is a characteristic quantity for electromagnetic calorimeters
c. is a characteristic quantity for hadronic calorimeters
d. characterises the distance that a quark radiates $1/e$ of its initial energy

Exercise 2

You can see below a typical event display from CMS (one of the four main experiments at the LHC).

1. Identify the type of particle or the specific species from the interactions you see in the fig. 0.2. **(2.5)**
2. For each line, starting from the top down to the last one in the bottom, argue about your choice in the previous question based on the information you got during the lectures. **(2.5)**

Exercise 3

1. Which reactions are possible, which are not, and why? For the reactions that are possible, indicate through which interaction they take place. For the reactions that are not possible, write down all the possible reasons. In case energy/momentum is the only reason, give the threshold energy for the incident particle (if you see a proton on the left hand side of the reaction consider it always as being stationary) needed for the reaction to take place.

a. $e^- + \mathrm{p} \rightarrow \mathrm{p} + \overline{\nu}_e$ **(0.5)**

b. $\Lambda \rightarrow \mathrm{p} + \pi^-$ **(0.5)**

c. $\mathrm{p} + \overline{\mathrm{p}} \rightarrow \pi^+ + \pi^- + \pi^0 + \pi^+ + \pi^-$ **(0.5)**

d. $\mathrm{p} + \overline{\nu}_e \rightarrow \tau^+ + \mathrm{e}^+ + \overline{\mathrm{p}}$ **(0.5)**

e. $\mathrm{p} + \overline{\nu}_\mu \rightarrow \mu^+ + \mathrm{n}$ **(0.5)**

The following numbers are given in natural units: $m_{\mathrm{e}^-} = m_{\mathrm{e}^+} = 0.511$ MeV, $m_{\mu^-} = m_{\mu^+} = 0.105$ GeV, $m_{\tau^-} = m_{\tau^+} = 1.777$ GeV, $m_{\pi^0} = 0.135$ GeV, $m_{\pi^-} = m_{\pi^+} = 0.139$ GeV, $m_{\mathrm{p}} = 0.938$ GeV, $m_{\mathrm{n}} = 0.939$ GeV, $m_\Lambda = 1.115$ GeV. For the purpose of this exercise, consider the neutrinos massless.

The quark composition where relevant: $\pi^+(u\overline{d})$, $\pi^0(u\overline{u}, d\overline{d})$, $\pi^-(d\overline{u})$, $\mathrm{p}(uud)$, $\mathrm{n}(udd)$, $\Lambda(uds)$.

2. Two particles, the first of spin 2 and third component -1 and the second with spin 1 and third component +1 form a composite system (consult Table **??**).

   a. What is the probability for each of the states of the composite system? **(1.5)**

   b. Which state is the most probable? **(0.5)**

   c. Show that the probabilities add up to unity. **(0.5)**

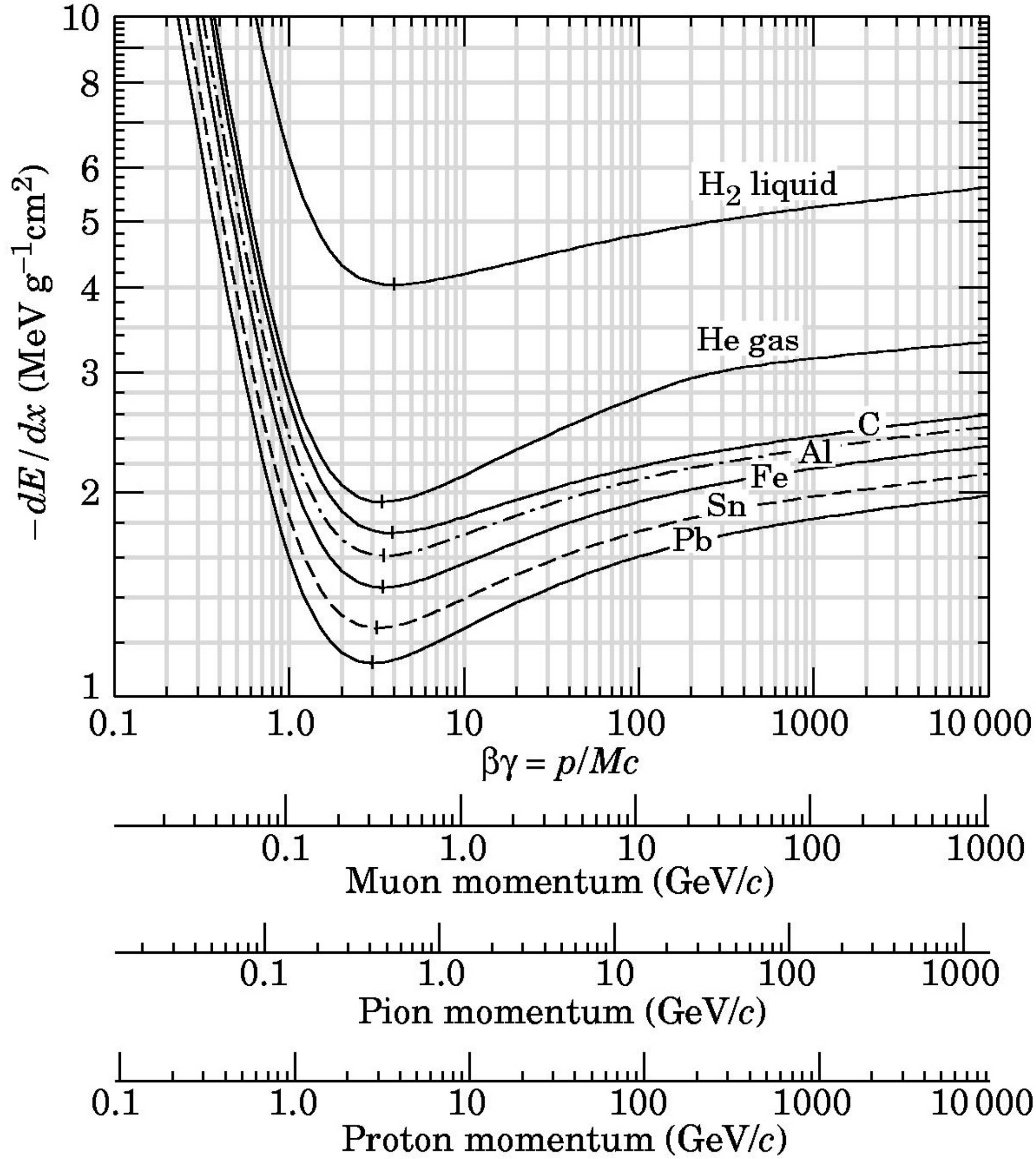
Exercise 4

1. A stationary observer on Earth observes spaceships A and B moving in the same direction toward the Earth. Spaceship A has speed 0.5c and spaceship B has speed 0.80c. What is the velocity of spaceship A according to an observer travelling on spaceship B? **(1)**
2. Show that light travels with the same speed regardless of the reference frame. **(1)**
3. Starting from energy-momentum 4-vector of the form

$$p^\mu = \begin{pmatrix} P_0 \\ \mathbf{P} \end{pmatrix}$$

   show that $E^2 = P^2c^2 + m^2c^4$. **1**

4. An electron of initial energy E and mass m scatters off elastically of a stationary electron. Express the angle with which the two electrons fly relative to each other in terms of E and m. **(2)**



10
8
6
5
4
3
2
1
$-dE/dx$ (MeV g^{-1}cm^2)
H_2 liquid
He gas
C
Al
Fe
Sn
Pb
0.1
1.0
10
100
1000
10 000
$\beta\gamma = p/Mc$
0.1
1.0
10
100
1000
Muon momentum (GeV/c)
0.1
1.0
10
100
1000
Pion momentum (GeV/c)
0.1
1.0
10
100
1000
10 000
Proton momentum (GeV/c)

Fig. 0.1

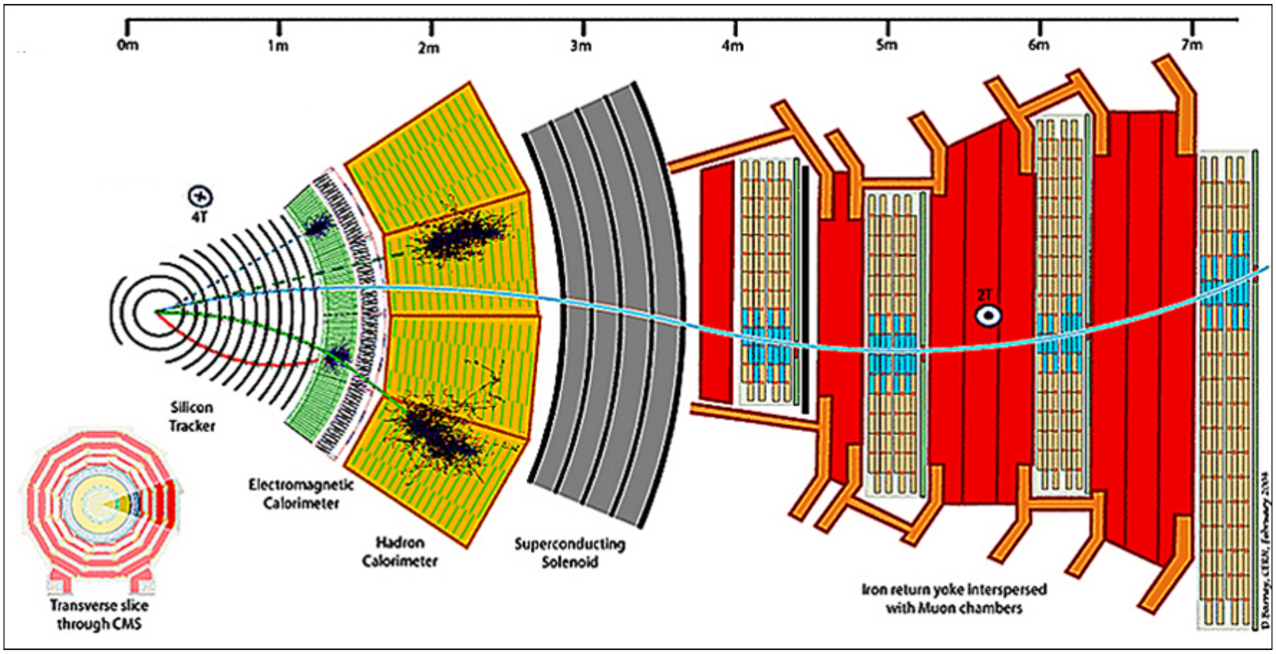

0m
1m
2m
3m
4m
5m
6m
7m
4T
2T
Silicon Tracker
Electromagnetic Calorimeter
Hadron Calorimeter
Superconducting Solenoid
Iron return yoke interspersed with Muon chambers
Transverse slice through CMS

Fig. 0.2

36. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for −8/15 read $-\sqrt{8/15}$.

Notation:

	J J …	
	M M …	
m_1 m_2		
m_1 m_2	Coefficients	
⋮ ⋮		

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^{\ell}_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^{\,j}_{m',m} = (-1)^{m-m'} d^{\,j}_{m,m'} = d^{\,j}_{-m,-m'}$$

1/2×1/2

m_1 m_2	1, +1
+1/2 +1/2	1

m_1 m_2	1, 0	0, 0
+1/2 −1/2	1/2	1/2
−1/2 +1/2	1/2	−1/2

m_1 m_2	1, −1
−1/2 −1/2	1

1×1/2

m_1 m_2	3/2, +3/2
+1 +1/2	1

m_1 m_2	3/2, +1/2	1/2, +1/2
+1 −1/2	1/3	2/3
0 +1/2	2/3	−1/3

m_1 m_2	3/2, −1/2	1/2, −1/2
0 −1/2	2/3	1/3
−1 +1/2	1/3	−2/3

m_1 m_2	3/2, −3/2
−1 −1/2	1

2×1/2

m_1 m_2	5/2, +5/2
+2 +1/2	1

m_1 m_2	5/2, +3/2	3/2, +3/2
+2 −1/2	1/5	4/5
+1 +1/2	4/5	−1/5

m_1 m_2	5/2, +1/2	3/2, +1/2
+1 −1/2	2/5	3/5
0 +1/2	3/5	−2/5

m_1 m_2	5/2, −1/2	3/2, −1/2
0 −1/2	3/5	2/5
−1 +1/2	2/5	−3/5

m_1 m_2	5/2, −3/2	3/2, −3/2
−1 −1/2	4/5	1/5
−2 +1/2	1/5	−4/5

m_1 m_2	5/2, −5/2
−2 −1/2	1

3/2×1/2

m_1 m_2	2, +2
+3/2 +1/2	1

m_1 m_2	2, +1	1, +1
+3/2 −1/2	1/4	3/4
+1/2 +1/2	3/4	−1/4

m_1 m_2	2, 0	1, 0
+1/2 −1/2	1/2	1/2
−1/2 +1/2	1/2	−1/2

m_1 m_2	2, −1	1, −1
−1/2 −1/2	3/4	1/4
−3/2 +1/2	1/4	−3/4

m_1 m_2	2, −2
−3/2 −1/2	1

2×1

m_1 m_2	3, +3
+2 +1	1

m_1 m_2	3, +2	2, +2
+2 0	1/3	2/3
+1 +1	2/3	−1/3

m_1 m_2	3, +1	2, +1	1, +1
+2 −1	1/15	1/3	3/5
+1 0	8/15	1/6	−3/10
0 +1	2/5	−1/2	1/10

m_1 m_2	3, 0	2, 0	1, 0
+1 −1	1/5	1/2	3/10
0 0	3/5	0	−2/5
−1 +1	1/5	−1/2	3/10

m_1 m_2	3, −1	2, −1	1, −1
0 −1	2/5	1/2	1/10
−1 0	8/15	−1/6	−3/10
−2 +1	1/15	−1/3	3/5

m_1 m_2	3, −2	2, −2
−1 −1	2/3	1/3
−2 0	1/3	−2/3

m_1 m_2	3, −3
−2 −1	1

1×1

m_1 m_2	2, +2
+1 +1	1

m_1 m_2	2, +1	1, +1
+1 0	1/2	1/2
0 +1	1/2	−1/2

m_1 m_2	2, 0	1, 0	0, 0
+1 −1	1/6	1/2	1/3
0 0	2/3	0	−1/3
−1 +1	1/6	−1/2	1/3

m_1 m_2	2, −1	1, −1
0 −1	1/2	1/2
−1 0	1/2	−1/2

m_1 m_2	2, −2
−1 −1	1

3/2×1

m_1 m_2	5/2, +5/2
+3/2 +1	1

m_1 m_2	5/2, +3/2	3/2, +3/2
+3/2 0	2/5	3/5
+1/2 +1	3/5	−2/5

m_1 m_2	5/2, +1/2	3/2, +1/2	1/2, +1/2
+3/2 −1	1/10	2/5	1/2
+1/2 0	3/5	1/15	−1/3
−1/2 +1	3/10	−8/15	1/6

m_1 m_2	5/2, −1/2	3/2, −1/2	1/2, −1/2
+1/2 −1	3/10	8/15	1/6
−1/2 0	3/5	−1/15	−1/3
−3/2 +1	1/10	−2/5	1/2

m_1 m_2	5/2, −3/2	3/2, −3/2
−1/2 −1	3/5	2/5
−3/2 0	2/5	−3/5

m_1 m_2	5/2, −5/2
−3/2 −1	1

3/2×3/2

m_1 m_2	3, +3
+3/2 +3/2	1

m_1 m_2	3, +2	2, +2
+3/2 +1/2	1/2	1/2
+1/2 +3/2	1/2	−1/2

m_1 m_2	3, +1	2, +1	1, +1
+3/2 −1/2	1/5	1/2	3/10
+1/2 +1/2	3/5	0	−2/5
−1/2 +3/2	1/5	−1/2	3/10

m_1 m_2	3, 0	2, 0	1, 0	0, 0
+3/2 −3/2	1/20	1/4	9/20	1/4
+1/2 −1/2	9/20	1/4	−1/20	−1/4
−1/2 +1/2	9/20	−1/4	−1/20	1/4
−3/2 +3/2	1/20	−1/4	9/20	−1/4

m_1 m_2	3, −1	2, −1	1, −1
+1/2 −3/2	1/5	1/2	3/10
−1/2 −1/2	3/5	0	−2/5
−3/2 +1/2	1/5	−1/2	3/10

m_1 m_2	3, −2	2, −2
−1/2 −3/2	1/2	1/2
−3/2 −1/2	1/2	−1/2

m_1 m_2	3, −3
−3/2 −3/2	1

2×3/2

m_1 m_2	7/2, +7/2
+2 +3/2	1

m_1 m_2	7/2, +5/2	5/2, +5/2
+2 +1/2	3/7	4/7
+1 +3/2	4/7	−3/7

m_1 m_2	7/2, +3/2	5/2, +3/2	3/2, +3/2
+2 −1/2	1/7	16/35	2/5
+1 +1/2	4/7	1/35	−2/5
0 +3/2	2/7	−18/35	1/5

m_1 m_2	7/2, +1/2	5/2, +1/2	3/2, +1/2	1/2, +1/2
+2 −3/2	1/35	6/35	2/5	2/5
+1 −1/2	12/35	5/14	0	−3/10
0 +1/2	18/35	−3/35	−1/5	1/5
−1 +3/2	4/35	−27/70	2/5	−1/10

m_1 m_2	7/2, −1/2	5/2, −1/2	3/2, −1/2	1/2, −1/2
+1 −3/2	4/35	27/70	2/5	1/10
0 −1/2	18/35	3/35	−1/5	−1/5
−1 +1/2	12/35	−5/14	0	3/10
−2 +3/2	1/35	−6/35	2/5	−2/5

m_1 m_2	7/2, −3/2	5/2, −3/2	3/2, −3/2
0 −3/2	2/7	18/35	1/5
−1 −1/2	4/7	−1/35	−2/5
−2 +1/2	1/7	−16/35	2/5

m_1 m_2	7/2, −5/2	5/2, −5/2
−1 −3/2	4/7	3/7
−2 −1/2	3/7	−4/7

m_1 m_2	7/2, −7/2
−2 −3/2	1

2×2

m_1 m_2	4, +4
+2 +2	1

m_1 m_2	4, +3	3, +3
+2 +1	1/2	1/2
+1 +2	1/2	−1/2

m_1 m_2	4, +2	3, +2	2, +2
+2 0	3/14	1/2	2/7
+1 +1	4/7	0	−3/7
0 +2	3/14	−1/2	2/7

m_1 m_2	4, +1	3, +1	2, +1	1, +1
+2 −1	1/14	3/10	3/7	1/5
+1 0	3/7	1/5	−1/14	−3/10
0 +1	3/7	−1/5	−1/14	3/10
−1 +2	1/14	−3/10	3/7	−1/5

m_1 m_2	4, 0	3, 0	2, 0	1, 0	0, 0
+2 −2	1/70	1/10	2/7	2/5	1/5
+1 −1	8/35	2/5	1/14	−1/10	−1/5
0 0	18/35	0	−2/7	0	1/5
−1 +1	8/35	−2/5	1/14	1/10	−1/5
−2 +2	1/70	−1/10	2/7	−2/5	1/5

m_1 m_2	4, −1	3, −1	2, −1	1, −1
+1 −2	1/14	3/10	3/7	1/5
0 −1	3/7	1/5	−1/14	−3/10
−1 0	3/7	−1/5	−1/14	3/10
−2 +1	1/14	−3/10	3/7	−1/5

m_1 m_2	4, −2	3, −2	2, −2
0 −2	3/14	1/2	2/7
−1 −1	4/7	0	−3/7
−2 0	3/14	−1/2	2/7

m_1 m_2	4, −3	3, −3
−1 −2	1/2	1/2
−2 −1	1/2	−1/2

m_1 m_2	4, −4
−2 −2	1

$$d^{1}_{0,0} = \cos\theta \qquad d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^{1}_{1,1} = \frac{1+\cos\theta}{2}$$

$$d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2} \qquad d^{1}_{1,0} = -\frac{\sin\theta}{\sqrt{2}}$$

$$d^{1}_{1,-1} = \frac{1-\cos\theta}{2}$$

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\frac{1+\cos\theta}{2}\sin\frac{\theta}{2} \qquad d^{2}_{2,2} = \Big(\frac{1+\cos\theta}{2}\Big)^2$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\frac{1-\cos\theta}{2}\cos\frac{\theta}{2} \qquad d^{2}_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2} \qquad d^{2}_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta \qquad d^{2}_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2} \qquad d^{2}_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta \qquad d^{2}_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2} \qquad d^{2}_{2,-2} = \Big(\frac{1-\cos\theta}{2}\Big)^2 \qquad d^{2}_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1) \qquad d^{2}_{0,0} = \Big(\frac{3}{2}\cos^2\theta-\frac{1}{2}\Big)$$

Figure 36.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974).