Department of Mathematics, Faculty of Science, UU.
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In 2009-2010, the course WISB243 was given by Dr. M. Crainic.

Inleiding Topologie (WISB243)
21-04-2010

During the exam, you may use the lecture notes.
Important: motivate/proce your answers to the questions. When making pictures, try to make them as clear as possible. When using a result from the lecture notes, please give a clear reference.

Question 1

Let X be the (interior of an) open triangle, as drawn in the picture (the edges are not part of X!), viewed as a topological space with the topology induced from $\mathbb{R}^2$. Let $A \subset X$ be the open disk drawn in the picture (tangent to the edges of the closed triangle). Compute the closure and the boundary of X. *(1 point)*

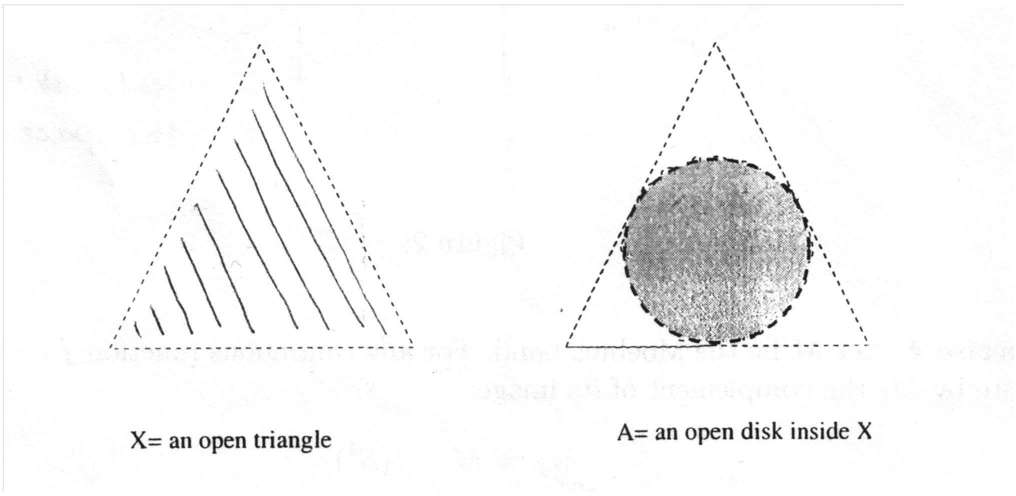

Figure 1: x and $A \subset X$

Question 2

Let X be obtained by taking two disjoint copies of the interval $[0, 2]$ (with the Euclidean topology) and gluing each t in the first copy with the corresponding t in the second copy, for all $t \in [0, 2]$ different from the middle point. Explicitely, one may take the space

$$Y = [0, 2] \times 0 \cup [0, 2] \times 1 \subset \mathbb{R}^2$$

with the topology induced from the Euclidean topology, and X is the space obtained from Y by gluing $(t, 0)$ to $(t, 1)$ for all $t \in [0, 2], t \neq 1$. We endow X with the quotient topology.

a) Is X Hausdorff? But connected? But compact? *(1.5 point)*

b) Can you find A, $B \subset X$ which, with the topology induced from X, are compact, but such that $A \cap B$ is not compact? *(1 point)*

c) Show that X can also be obtained as a quotient of the circle S^1. *(0.5 point)*

Question 3

Let X, Y and Z be the spaced drawn in .

a) Show that any two of them are not homeomorphic. *(1.5 point)*

b) Compute their one-point compactifications X^+, Y^+ and Z^+. *(1 point)*

c) Which two of the spaced X^+, Y^+ and Z^+ are homeomorphic and which are not? *(1 point)*

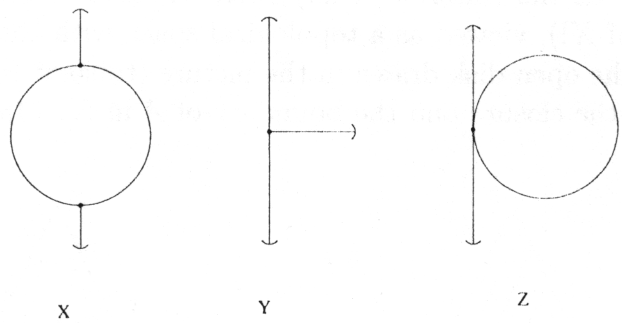

Figure 2: X, Y and Z

Question 4

lET M be the Moebius band. For any continuous function $F : S^1 \to M$ we denote by M_f the complement of its image:

$$M_f := M - f(S^1)$$

and we denote by M_f^+ the one-point compactification of M_f.

a) Show that for any f, M_f is open in M, it is locally compact but not compact. *(1 point)*

b) Find an example of f such that M_f^+ is homeomorphic to D^2. Then one for which it is homeomorphic to S^2. And then one for $\mathbb{P}^2$. *(1.5 point)*