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In 2006/2007, the course WISB272 was given by Alexander Gnedin.

Speltheorie (WISB272)
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Question 1

A noncooperative game is given by the bimatrix

$$\begin{pmatrix} (1,3) & (1,3) & (0,2) \\ (2,0) & (1,1) & (3,2) \\ (2,2) & (2,3) & (1,1) \end{pmatrix}$$

a) Determine the safety levels and maxmin strategies for both players. *(10 points)*

b) Find all Nash equilibria. *(10 points)*

Question 2 ***(10 points)***

Consider the Cournot duopoly model. Suppose the cost of producing q_1 units of some product is equal to $2q_1 + 1$ for firm F1; and the cost of producing q_2 units of the same product is equal to $q_2 + 2$ for firm F2. The price function is given by the formula $P(q_1, q_2) = (24 - q_1 - q_2)_+$. Determine the production levels and profits of the firms F1 and F2 at the equilibrium.

Question 3

A cooperative game with transferable utility (TU) is given by the bimatrix

$$\begin{pmatrix} (2,1) & (4,2) & (1,1) \\ (4,2) & (-1,1) & (3,2) \\ (0,0) & (-1,1) & (-1,2) \end{pmatrix}$$

a) Compute the TU-values of both players. *(10 points)*

b) Find the optimal threat strategies. *(5 points)*

c) Determine the associated side payment. *(5 points)*

Question 4

A cooperative game with nontransferable utility (NTU) is given by the bimatrix

$$\begin{pmatrix} (2,1) & (3,2) \\ (2,3) & (1,2) \end{pmatrix}$$

a) Determine the set of feasible payoff vectors. *(5 points)*

b) Determine the set of Pareto optimal payoff vectors. *(5 points)*

c) Assuming that $(u^*, v^*) = (2,2)$ is the disagreement point, find the NTU-solution of the game. *(10 points)*

Question 5 *(10 points)*

Compute the Shapley value of the n-person game ($n \geq 3$) with characteristic function

$$v(S) = \begin{cases} \#S & \text{if } \#(S \cap \{1,2,3\}) > 0 \\ 0 & \text{otherwise} \end{cases}$$

Question 6

Consider the 3-person game in coalitional form with characteristic function

$$\begin{array}{rrrr} v(\varnothing) = 0 & v(\{1\}) = 1 & v(\{2\}) = 2 & v(\{3\}) = 0 \\ & v(\{1,2\}) = 4 & v(\{1,3\}) = 2 & v(\{2,3\}) = 3 \\ & & & v(\{1,2,3\}) = 7 \end{array}$$

a) Find the set of imputations and find the core of the game. (Either draw the core graphically or be fairly explicit in your description.) *(7 points)*

b) Find the nucleolus. *(7 points)*

c) Compute the Shapley value. *(6 points)*