1ST EXAM 'INLEIDING IN DE GETALTHEORIE'

Tuesday, 27th September 2016, 9 am - 10 am

Question 1
Find all $x \in \mathbb{Z}$ such that

$$x \equiv 1 \bmod 2, \quad x \equiv 3 \bmod 5, \quad \text{and} \quad x \equiv 5 \bmod 7.$$

Question 2
Let $k \in \mathbb{N}$. We define $\sigma_k(n) := \sum_{d|n} d^k$. Show that $\sigma_k(n)$ is a multiplicative function, i.e. $\sigma_k(mn) = \sigma_k(m)\sigma_k(n)$ for natural numbers m, n with $\gcd(m, n) = 1$.

Question 3
Let $k \geq 1$. Show that there is a natural number x such that all of the numbers $x, x+1, x+2, \ldots, x+k$ have a non-trivial fourth power divisor, i.e. such that for every $0 \leq i \leq k$ there is an integer $d_i \geq 2$ with $d_i^4 | (x+i)$.

Question 4
Let $n \geq 2$. Show that

$$\sum_{\substack{m=1 \\ \gcd(m,n)=1}}^{n-1} m = \frac{1}{2} n \phi(n).$$

Note: Only pen and paper are allowed for the exam!

Date: 27th September 2016.