

2ND EXAM 'INLEIDING IN DE GETALTHEORIE'

Tuesday, 11th October 2016, 9 am - 10 am

Question 1

Compute the following symbols

$$\left(\frac{71}{97}\right), \quad \left(\frac{53}{97}\right), \quad \left(\frac{137}{227}\right), \quad \left(\frac{10}{5}\right)$$

Question 2

Let p be an odd prime number. In the lectures we have seen that $\left(\frac{-1}{p}\right) = 1$ if and only if $p \equiv 1 \pmod{4}$. Characterize in the same way all prime numbers with $\left(\frac{5}{p}\right) = 1$.

Question 3

Let p be an odd prime number and q the smallest quadratic non residue modulo p . More precisely, let q be the smallest natural number which is not a quadratic residue modulo p . Prove that q is a prime number.

Question 4

Show that the sequence $n^5 - n + 3$, with $n \in \mathbb{N}$ does not contain any squares.

Note: A simple non-programmable calculator is allowed for the exam. (If you don't have one, don't worry, you don't necessarily need it.)