FINAL EXAM 'INLEIDING IN DE GETALTHEORIE'

Thursday, 5th January 2016, 13.30 pm - 16.30 pm

Question 1
a) Find the continued fraction expansion of $\sqrt{33}$.
b) What number has the continued fraction expansion

$$[5, 1, 4, 1, 10, 1, 4, 1, 10, ...] \quad ?$$

Question 2
Show that there is an infinite number of primes of the form $p = 6m + 1$ with $m \in \mathbb{N}$. (Hint: consider expressions of the form $12x^2 + 1$.)

Question 3
Let $a \in \mathbb{N}$. Assume that $\left(\frac{a}{p}\right) = 1$ for every odd prime number $p \nmid a$. Show that then a has to be a square number.

Question 4
Give a proof of the identity

$$\sum_{d|n} (-1)^{n/d} \phi(d) = \begin{cases} 0 & \text{if} \quad n \in \mathbb{N} \text{ is even} \\ -n & \text{if} \quad n \in \mathbb{N} \text{ is odd.} \end{cases}$$

Question 5
Is there a natural number $n \in \mathbb{N}$ such that $d(n) = 7$ where $d(n)$ is the divisor function? Is there a natural numer n such that $\phi(n) = 7$?

Question 6
A Pythagorean triangle is a triangle with one right angle and such that all the sides have integer length. We say that a pythagorean triangle has consecutive legs, if the difference between the two shortest sides is exactly equal to one. Find at least 2 different pythagorean triangles with consecutive legs and show how pythagorean triangles with consecutive legs are related to solutions of a certain Pell's equation.

Note: A simple non-programmable calculator is allowed for the exam.

Date: 5th January 2016.