

First exam – Elementaire Getaltheorie

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In all problems write your solution in detail. Each step has to be proven or cited from class.

Problem 1 (10 points). *For the following two equations, decide whether they have solutions with $x, y \in \mathbb{Z}$. If yes, give two different pairs (x, y) of solutions.*

(a) $447x + 408y = -3$

(b) $447x + 408y = 7$

Decide furthermore if the system of congruences

$$a \equiv -3 \pmod{447}$$

$$a \equiv 7 \pmod{408}$$

has a solution $a \in \mathbb{Z}$ and if yes, give such a solution.

Problem 2 (10 points). *Let a be an arbitrary integer.*

(a) *Compute the remainder of a^{36} if we divide by 36.*

(b) *Show that $a^{36} - 1$ is not a prime number.*

Problem 3 (10 points). *Recall that the sum of positive divisors $\sigma(n)$ of a natural number n with prime factorization $p_1^{k_1} \cdots p_r^{k_r}$ with $p_1 < \cdots < p_r$ equals*

$$\prod_{i=1}^r \frac{p_i^{k_i+1} - 1}{p_i - 1}.$$

Give a similar formula for

$$\sum_{0 < d|n} d^2.$$