2ND EXAM 'INLEIDING IN DE GETALTHEORIE'

Tuesday, 9th October 2018, 9 am - 10 am

Question 1 (4 points)
a) Find the orders of 2, 3 and 5 modulo 23.
b) Find all primitive roots modulo 7, 14 and 49.

Question 2 (4 points)
Compute the following symbols

$$\left(\frac{313}{367}\right), \quad \left(\frac{367}{401}\right), \quad \left(\frac{401}{313}\right), \quad \left(\frac{2}{313}\right).$$

Question 3 (4 points)
Let n be a natural number and assume that there is no odd prime number p with $p^2 \mid n$. We write $\nu(n)$ for the number of residue classes x modulo n with $x^2 \equiv -1 \bmod n$. Let S be the set of odd prime divisors of n.
a) Assume that $4 \mid n$ or that there is a prime number $p \in S$ with $p \equiv 3 \bmod 4$. Deduce that $\nu(n) = 0$.
b) Assume that $4 \nmid n$ and that $p \equiv 1 \bmod 4$ for all primes p contained in S. Show that in this case we have

$$\nu(n) = 2^{|S|},$$

where we write $|S|$ for the cardinality of the set S.

Question 4 (4 points)
Let p be an odd prime number and k a natural number. Show that

$$1^k + 2^k + \ldots + (p-1)^k \equiv \begin{cases} 0 \mod p & \text{if } \gcd(p-1, k) = 1 \\ -1 \mod p & \text{if } p-1 \mid k \end{cases}$$

(Note: the first statement even holds under the stronger assumption $p-1 \nmid k$)

Note: A simple non-programmable calculator is allowed for the exam.

Date: 9th October 2018.