Mastermath midterm examination Parallel Algorithms.

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Each of the four questions is worth 10 points. Total time 90 minutes.

1. Explain the four parameters of the BSP model. Which parameters are relevant for the design of a BSP algorithm?

2. Let $\mathbf{x}$ be an array of length n containing numerical values x_i, where $0 \leq i < n$. All values are different. Our aim is to find the runner-up in parallel, using a suitable data distribution, and p processors. The runner-up is defined as the second-largest data element. Give an efficient BSP algorithm for processor $P(s)$ for this computation in the notation we have learned. Analyse the BSP cost.

3. Assume we have a vector $\mathbf{x}$ of n integers, with n even, which we want to sort using mergesort on two processors. Assume that the vector is distributed by the block distribution. On output it also has to be in this distribution. Design a parallel mergesort algorithm and analyse its BSP cost. It is sufficient to describe the algorithm in words; no need for a program text. Extend your algorithm to an algorithm for four processors.

4. Let A be a dense $n \times n$ matrix, with elements a_{ij} where $0 \leq i, j < n$. Let B be the $k \times k$ matrix

$$B = \begin{bmatrix} I_{k/2} & I_{k/2} \\ I_{k/2} & -I_{k/2} \end{bmatrix},$$

where $I_{k/2}$ is the identity matrix of size $k/2 \times k/2$. Design a parallel algorithm for multiplying the matrix $B \otimes A$ and a vector $\mathbf{x}$ of length kn using p processors and formulate it as an algorithm for processor $P(s)$ in the notation we have learned. Assume that every processor has

a copy of the matrix A. Also assume that $k \bmod 2p = 0$. Choose a suitable distribution ϕ for the input vector $\mathbf{x}$. The output vector must be obtained in the same distribution as the input vector. Analyse the BSP cost of your algorithm.