

## Parallel Algorithms (WISM459) October 11, 2006

*Each of the five questions is worth 10 points. Total time 45 minutes. Note that this exam has 2 pages!*

### Question 1

Explain the architecture of a BSP computer.

### Question 2

50 processors of a parallel computer are matched in pairs  $(P(s), P(s+1))$ , with  $s$  even. Processor  $P(s)$  sends  $s$  data words to processor  $P(s+1)$ , for even  $s$ . What is the value  $h$  of the  $h$ -relation defined by this communication pattern?

### Question 3

Let  $\mathbf{x}$  and  $\mathbf{y}$  be given vectors of length  $n$  which are distributed over  $p$  processors, with  $n \bmod p = 0$ ;  $\mathbf{x}$  is distributed by the cyclic distribution, and  $\mathbf{y}$  by the block distribution. Give an efficient BSP algorithm for processor  $P(s)$  (in the notation we learned) for the computation of the vector  $\mathbf{z} = \mathbf{x} + \mathbf{y}$ . Analyse its BSP cost. You are free to choose the output distribution.

### Question 4

Let  $p, n$  be integers, with  $2 \leq p \leq n$ . Define a permutation  $\sigma$  by  $\sigma(i) = n - 1 - i$ , for  $0 \leq i < n$ . What is the exact communication cost of permuting a block distributed vector  $\mathbf{x}$  by  $\sigma$ , i.e., assigning  $y_{\sigma(i)} = x_i$ ? The length of the input and output vectors is  $n$ .

### Question 5

We want to evaluate a polynomial

$$f(x) = \sum_{i=0}^{n-1} a_i x^i$$

in parallel using  $p$  processors. Here,  $x$  is a given real number and  $a_0, \dots, a_{n-1}$  are given real coefficients. Sequentially, this is usually done by applying Horner's rule:

$$f(x) = a_0 + x(a_1 + x(a_2 + \dots + x(a_{n-2} + x(a_{n-1})) \dots)).$$

Assume that  $x$  is known at  $P(0)$ , and that the coefficients are already distributed by the block distribution over  $p$  processors, where  $n \bmod p = 0$ . Give an efficient BSP algorithm for processor  $P(s)$  for the computation of  $f(x)$ . The output  $f(x)$  should become known at  $P(0)$ . Analyse the BSP cost.