

Structure of Matter (NS-266B)

Resit 2015 – solutions

Part I – Condensed Matter

Exercise 1.1 (22 points)

a) From the real space picture it is clear that this is a (binary) smectic liquid crystal phase, because it is mentioned that there is diffusion (liquid like) behavior in the layers, and the ordering (see also the scattering with multiple order Bragg peaks) perpendicular to the layers is 1D solid (crystal) like. (4 points using the definition of a rod-like smectic phase).

b) The scattering vector lengths at which the 1D order is found are multiples of $q = 2\pi/(l + d)$, where l is the rod length and d is the sphere diameter (it can be seen that roughly one sphere diameter is in between two layers of rods, but other values around this were OK as well), the units are in $(2\pi/(2+0.2)) \mu\text{m}^{-1}$ (no units: -1 point). The larger q value associated with the distance between the rods is roughly the same as the diameter of the rods which is at: $q = (2\pi/(0.2)) \mu\text{m}^{-1}$ here only one “smeared out” peak as the order is not crystalline, but liquid-like! (total 6 points, 3 for each value).

c) Now that only the spheres scatter, their order will only show up in the scattering pattern. As their layering is similar (and dictated) by that of the rods, the scattering pattern will be more or less similar (with similarly spaced) peaks as that shown for the rods. Most likely the ordered peaks perpendicular to the 1D crystalline order are missing as the order of the spheres in the plane is most likely less well-defined (see image) as that of the rods, but this was not essential to mention for the full points: 6 points.

d) The pair distribution function of the rods in the plane looks essentially like that of a 2D $g(r)$ of a liquid of 2D disks or spheres as shown on page 24 of the Sidebottom book. The first peak is at the diameter of the rods (200 nm) and goes up very steep as these are hard particles. Subsequent peaks (at multiples of the diameter) oscillate roughly around 1 and become less high. (points: 6, -1 point for incorrect position and or not mentioned peaks around 1, -1 for values >0 for distances $<$ diameter).

Exercise 1.2 (13 points)

a) a, e, h, i and j . Smallest area, correct stoichiometry (ABC_2 atoms in cell) and possible to fill 2D space by translations only. (5 points, -1 point if incorrect).

b) c , same as requirements as for a primitive cell, except larger area (3 points, -1 if incorrect).

c) Define where the origin is, use two basis vectors (here e.g. a_1 and a_2) and give all atom positions that are in the primitive cell with respect to these vectors: basis set. *E.g. for cell a: Origin in most left atom B: (0,0) Other B atoms: ($1/2, 1/2$), (1,0), (1.5, $1/2$). Atoms A: ($1/2, 0$), (1, $1/2$), Atoms C: ($1/4, 0$), (1, $1/2$). Etc.* (5 points, -1 point if not all points in cell are given (e.g. all points that are ‘half’ in or less need to be given in coordinates, -1 if no origin given).

Exercise 1.3 (25 points)

a) An intrinsic semiconductor, like Si, is characterized by the fact that their small energy gap

allows a small concentration of electrons to be *thermally* excited from a filled valence band into the empty conduction band. Thus for an intrinsic semiconductor the number of electrons and holes is the same. In order to alter this distribution and make an extrinsic semiconductor with a majority of holes (*p*-type) or conduction electrons (*n*-type, impurity e.g As implantation), impurities that have with respect to the silicon base atoms an electron to many (creating extra conduction electrons) or to few (creating extra holes, e.g. by B implantation). Points: 5

b) See also the last part of a) Bringing a *n*-type and *p*-type doped material together creates a *pn*-junction which will spontaneously create a voltage difference between these regions and with which electron currents can be rectified (as in a diode). In a more complex geometry a *pn*-junction can be used to switch currents on or off with an applied potential: a transistor.

c) The temperature dependence of a metallic conductor and that of a semiconductor like Si are opposite in that with increasing temperature the resistivity of metals increases (e.g. by increased scattering of phonons of the conduction electrons), while that of semiconductors decreases as the increased resistance by phonon scattering is (far) offset by an (exponential) increase in the number of conduction electrons that can be excited from the *v*-band to the *c*-band. (5 points)

d) The derivation of this exponential increase in conduction electrons makes use of the small band gap and the Fermi-Dirac distribution which gives the probability of finding electrons at energy *E* times the density of states and integrating this from energies of the lower edge of the conduction band upwards. For symbols of FD distribution (3 points) (See Sidebottom page 234, for full 7 points):

The number of electrons in the conduction band is given as,

$$N = \int_{E_C}^{E_T} P_{FD}(E)g(E)dE = \int_{E_C}^{E_T} \left\{ \frac{1}{\exp[(E - E_F)/k_B T] + 1} \right\} g(E)dE, \quad (13.31)$$

where E_C is the lower edge of the conduction band. Since $P_{FD}(E)$ decreases very rapidly above E_F at any meaningful temperatures, we can (1) replace the density of states by its quadratic form valid near the bottom of a band, and (2) extend the upper limit of integration to infinity. Furthermore, for a typical intrinsic semiconductor whose energy gap is around 1 eV, the exponential in the denominator of $P_{FD}(E)$ is large compared to unity, and we can rewrite Eq. (13.31) as,

$$N = \left(\frac{V}{2\pi^2} \right) \left(\frac{2m_C^*}{\hbar^2} \right)^{3/2} \int_{E_C}^{\infty} (E - E_C)^{1/2} \exp[-(E - E_F)/k_B T] dE,$$

or,

$$n = N/V = 2 \left(\frac{m_C^* k_B T}{2\pi\hbar^2} \right)^{3/2} \exp[(E_F - E_C)/k_B T]. \quad (13.32)$$

Part II – Subatomic Physics

Exercise 2.1: Multiple choice questions

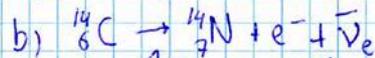
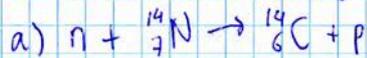
1 B
2 D
3 D
4 B
5 A

6 C
7 B
8 D
9 B
10 A

Exercise 2.2: Radiocarbon dating

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2.2 Radiocarbon dating



c) ~~$R = \frac{N_{14C}}{N_C}$~~ $R = \frac{N_{14C}}{N_C} = 1,33 \cdot 10^{-12}$

A gram carbon contains $N_C = \frac{1}{12,011} \cdot 6,022 \cdot 10^{23} = 5,01 \cdot 10^{22}$ C atoms

and $N_{14C} = RN_C = 1,33 \cdot 10^{-12} \cdot 5,01 \cdot 10^{22} = 6,66 \cdot 10^{10}$ ${}^{14}\text{C}$ atoms

The activity is defined as $A(t) = -\frac{dN(t)}{dt} = \lambda N(t)e^{-\lambda t} = A(0)e^{-\lambda t}$

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{5730\text{y}} = \frac{\ln 2}{5730 \cdot 365,25 \cdot 24 \cdot 3600\text{s}} = \frac{\ln 2}{1,808 \cdot 10^{11}} = 3,83 \cdot 10^{-12} \text{ s}^{-1}$$

The activity of atmospheric carbon per gram of carbon is then

$$A(0) = \lambda N(0) = \lambda N_{14C} = 3,83 \cdot 10^{-12} \cdot 6,66 \cdot 10^{10} \text{ s}^{-1} = 0,256 \text{ Bq}$$

d) Look at the ratio of activities:

$$\frac{A(t)}{A(0)} = \frac{\lambda N(t)}{\lambda N(0)} = e^{-\lambda t} \rightarrow t = \frac{\log\left(\frac{A(t)}{A(0)}\right)}{-\lambda}$$

In this case, $A(0) = 0,5 \cdot 0,256 \text{ Bq} = 0,128 \text{ Bq}$

$$A(t) = 174 \text{ h}^{-1} = \frac{174}{3600} \text{ s}^{-1} = 0,0483 \text{ Bq}$$

$$t = \frac{\log\left(\frac{0,0483}{0,128}\right)}{-\frac{\ln 2}{5730\text{y}}} \approx 8044\text{y}$$

Points

a, b) correct answer: 2 points

c) +1 Number of C atoms in 1g, N_C

+1 Number of ${}^{14}\text{C}$ atoms in 1g, N_{14C}

+2 Correct usage of the activity $A(t) = -\frac{dN(t)}{dt}$, $A(0) = \lambda N_{14C}$

+1 Calculate λ

+1 Correct final answer in Bq

d) +1 Activity of specimen when it was alive, $A(0)$

+1 Activity of specimen now, $A(t)$

+1 Ratio of activities $t = \frac{\log(A(t)/A(0))}{-\lambda}$

+2 Correct expression of $t = \frac{\log(A(t)/A(0))}{-\lambda}$

+1 Correct final answer

Exercise 2.3: Particle decays

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2.3 Particle decays and conservation laws

a) $\pi^0 \rightarrow \gamma + \gamma$ is an electromagnetic process

$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ is a weak process

Since the electromagnetic force is stronger, the π^0 decays faster, which means the π^- has a longer lifetime

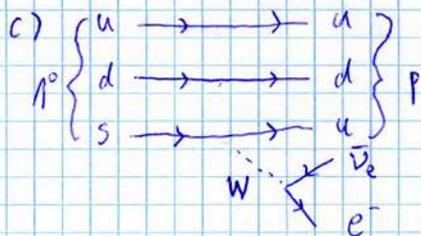
b) not allowed, baryon number not conserved

2. not allowed, muon lepton number not conserved

3. allowed, strangeness not conserved but that is allowed in the weak int.

4. allowed

5. allowed



Points

a) +2 for correct answer, +2 for motivation

b) +1 for correctly saying if the process is allowed or not

+1 for checking the conservation laws

c) +1 correct quark contents

+3 Feynman diagram

-1 no arrows

-1 wrong force

other mistakes also give -1.