Subatomic Physics
Exam 2018

10 April 2018

Abstract

Time: 13:30 – 16:30 (3 hours) – Please do not leave before 14:15.
The exam consists of two parts:
Part I tests knowledge in ten multiple-choice questions.
Part II consists of four open exercises.
The maximal number of points is indicated for each exercise.
Answer each of the exercises on a separate piece of paper.
Write your name and student number on each page.
Do not give final answers only, explain your reasoning (short) and give full calculations. A simple calculator use is allowed (no programmable).
No mobile/smart phone!
Good luck!

1 Part 1 - Multiple Choice questions (20 points)

1. Which type of particle has 0 spin?
   A. Fermion
   B. Higgs
   C. Boson
   D. Lepton
2. The Rutherford's scattering law is true when the α-particle:
   A. penetrates the nucleus.
   B. undergoes a deep inelastic scattering.
   C. has a head-on collision on the gold nucleus.
   D. is absorbed by the gold nucleus.
3. The rest energy of an electron is of the order of:
   A. eV
   B. keV
   C. MeV
   D. GeV
4. Rutherford scattering does not depend on the ... of the incoming particle.
   A. electric charge
   B. kinetic energy
   C. impact parameter
   D. spin
5. What was the first major argument for the existence of the quantized photon:
   A. The observation of X-rays.
   B. The process of particle annihilation.
   C. An explanation of black-body radiation.
   D. The continuous spectrum of β-decay.
6. Via which decay will $^{14}_{6}C$ decay?
   A. α decay
   B. β^- decay
   C. β^+ decay
   D. γ decay
7. Sort the four fundamental forces from long to short range.
   A. Gravitational force - Electromagnetic force - Weak force - Strong force
   B. Electromagnetic force - Gravitational force - Strong force - Weak force
   C. Gravitational force - Electromagnetic force - Strong force - Weak force
   D. Strong force - Weak force - Gravitational force - Electromagnetic force
8. Which particles do not interact via the strong interaction?
   A. Baryons

B. Leptons

C. Mesons

D. Fermions

9. Which of the following nuclei has the highest binding energy per nucleon?

A. ${}^{56}_{26}\mathrm{Fe}$

B. ${}^{14}_{7}\mathrm{N}$

C. ${}^{16}_{8}\mathrm{O}$

D. ${}^{238}_{92}\mathrm{U}$

10. Magnetic resonance imaging (MRI) is based on the

A. electron spin

B. isospin

C. Lorentz force

D. nuclear spin

2 Part 2 - Open questions

2.1 Rutherford Scattering (20 points)

Equation 1 describes Rutherford Scattering.

$$b = \frac{1}{4\pi\epsilon_0}\frac{ZZ'e^2}{2E}\cot\frac{\theta}{2} \tag{1}$$

a) (**5 points**) Draw a diagram showing the path of a positively charged point particle undergoing scattering at $\theta = 120°$ from

  i) an uncharged hard sphere,

  ii) an identical sphere with a uniform positive charge.

  Label the important lengths and angles, defining any symbols used.

b) (**5 points**) Describe the conditions for which Equation 1 holds and how the breakdown of this relationship can be useful.

c) (**5 points**) Consider an experiment involving 10 MeV protons scattering from a thin foil made of $^{208}_{82}$Pb. Calculate the scattering cross section for angles $> 20°$, considering the nucleus as

  i) an uncharged hard sphere,

  ii) an identical sphere with a uniform positive charge.

d) (**5 points**) The differential cross section $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$ is defined by equation 2, where $\Delta\sigma$ is the transverse cross sectional area presented for scattering into the surface element $\mathrm{d}\Omega$ at θ, ϕ.

$$\Delta\sigma(\theta,\phi) = -\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\theta,\phi)\sin\theta\mathrm{d}\theta\mathrm{d}\phi \tag{2}$$

  Derive a general expression for the differential cross section. 'General' here means that the expression could be used to calculate the form of $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\theta)$ when the relationship between b and θ is known.

Useful values: $k_e e^2 = 1.440$ eV nm, $r_0 = 1.25$ fm.

2.2 Direct detection of dark matter (20 points)

The Standard Model has demonstrated huge successes in providing experimental predictions, however, it only accounts for 5% of the content of the Universe. We know from cosmological observations that about 27% is dark matter. The generic properties of dark matter are understood, but we have yet to directly detect it or find the particle(s) responsible for it.

One widely followed hypothesis is that dark matter are weakly-interacting massive particles (WIMP's, denoted with χ), distributed in a halo following a $1/r^2$ density distribution in our galaxy. The XENON experiment is one of the dedicated experiments trying to detect these WIMP's by looking for recoil energies of elastic scatterings between WIMP's and ^{131}Xe nucleus (or one of its constituents).

a) (**2 points**) A WIMP with mass m_χ scatters with a target nucleus (mass M_N) with a non-relativistic speed v. The scattering angle in the center-of-mass frame is given by θ. Give the expression for the recoil energy of the nucleus.

b) (**2 points**) Theories predict values for the WIMP of $m_\chi = 100$ GeV/c^2 and $v = 220$ km/s. What is the typical energy scale of which the XENON detector should be sensitive to detect WIMP's. Assume 1u $\approx$ 1 GeV/c^2

Assume we have N_{targets}, a flux of WIMP's traveling through our detector of size Φ, and a scattering cross section of $\sigma_{\text{N}} \equiv \sigma_{\text{N}}(\chi + N \to \chi + N)$.

c) (**2 points**) Give an expression for the number of events we expect.

Filling in some (optimistic) values here, we can expect 0.1 interactions/kg/year. Understanding all your background processes is therefore of extreme importance for these type of experiments.

One can shield the detector from external backgrounds like cosmic muons, however, internal background processes are more difficult. These are mainly coming from the contamination of the detector material with ^{238}U and ^{232}Th. These elements have been present since the formation of the Earth, and are still around because of their extremely large lifetime (several gigayears).

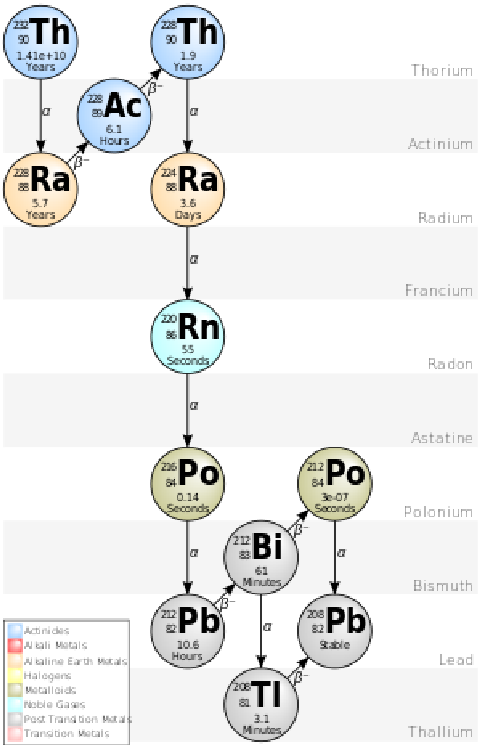

Figure 1: Decay chain of ^{232}Th.

d) (**2 points**) The decay chain of ^{232}Thorium is shown in Figure 1. Write down the equation for the rate of change of the ^{228}Ra isotope.

e) (**3 points**) Show that the activity ($A(t) = \lambda N(t)$) for ^{228}Ra is given by:

$$A_{^{228}\mathrm{Ra}}(t) = A_{^{232}\mathrm{Th}}(0)\frac{\lambda_{^{228}\mathrm{Ra}}}{\lambda_{^{228}\mathrm{Ra}} - \lambda_{^{232}\mathrm{Th}}}(e^{-\lambda_{^{232}\mathrm{Th}}t} - e^{-\lambda_{^{228}\mathrm{Ra}}t}) \tag{3}$$

f) (**2 points**) Simplify this expression using the lifetimes of ^{232}Th and ^{228}Ra shown in Figure 1.

g) (**3 points**) Sketch the activities $A_{^{228}\mathrm{Ra}}(t)$ and $A_{^{232}\mathrm{Th}}(t)$ as function of time. *Hint: look at timescales from* $t = 0$ *till* $t_{1/2}(^{228}Ra) \ll t \ll t_{1/2}(^{232}Th)$

h) (**2 points**) After some time all isotopes in the decay chain will be in a so-called 'secular equilibrium'. Look again at your answer for question d), how much ^{216}Po do we have if there is $N_{^{220}\mathrm{Rn}}$ at this moment? Assume t is long enough for secular equilibrium.

The used detectors for these experiments have been getting larger and more sensitive in recent years. There will however be a hard limit after which there is no point in making your dark matter detector larger anymore.

i) (**2 points**) Which Standard Model particle will be the cause of this irreducible background?

2.3 Conservation laws and Feynman diagrams (20 points)

Check the following particle reactions and decays for violation of the conservation of energy/mass, electric charge, baryon number, lepton number and strangeness number (use the enclosed tables) say whether they are allowed or forbidden and why:

a) (**2 points**) $\mu^- \to \overline{\nu_\mu} + e^- + \overline{\nu_e}$

b) (**2 points**) $\pi^0 + p \to n + p$

c) (**2 points**) $e^- + p \to n + \nu_e$

d) (**2 points**) $D^+ \to K^- + \pi^+ + \pi^+$

e) (**2 points**) $p \to p + K^- + K^+$

f) (**2 points**) $J/\Psi \to \mu^+ + \mu^-$

Write down the Feynman diagrams on quark level for the following particle reactions (for the quark content of each particle see the enclosed tables):

g) (**4 points**) $g + u \to b + \bar{b} + u$

h) (**4 points**) $K^+ \to \pi^+\pi^+\pi^-$

2.4 Pion-proton scattering (20 points)

Meson factories produce intense secondary beams of for example pions by striking a target with a primary proton beam. These secondary beams are used to study the interaction of pions with nuclei. Figure 2 shows the result of such an experiment, where a short-lived unknown particle "X" is produced:

$$\pi^+ + p \to X \to \pi^+ + p$$

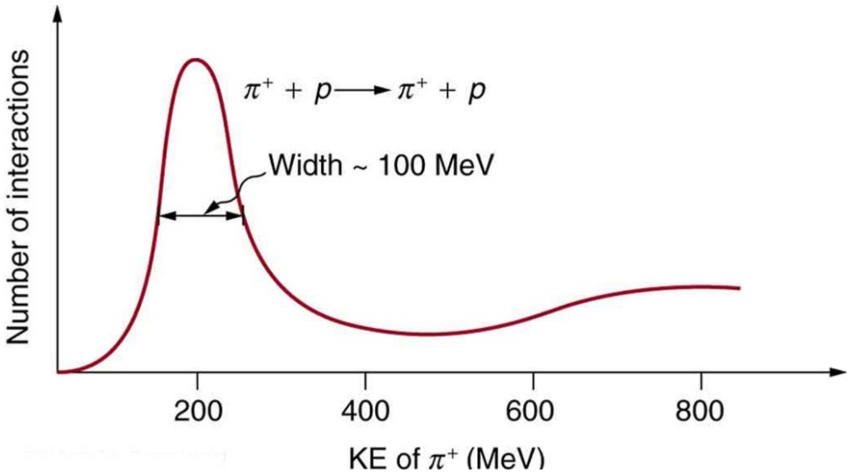

Figure 2: The probability of the interaction between a π^+ and a proton as a function of the kinetic π energy.

a) (**5 points**) The x-axis indicates the kinetic energy $E_{kin} = E_\pi - m_\pi^2 c^4$. Assume that the target proton is at rest.
What is the relationship between the invariant pion-proton mass m_{inv} and the kinetic energy of the pion?

b) (**5 points**) Deduce what kind of particle X is.

c) (**5 points**) Which interaction is responsible for the decay?

d) (**5 points**) Use Fig. 2 to derive the lifetime of the particle (tip: use the uncertainty relationship).

Table 12-11 Quark composition of selected hadrons

Baryons	Quarks	Mesons	Quarks
p	uud	π^+	$u\bar{d}$
n	udd	π^-	$\bar{u}d$
Λ^0	uds	K^+	$u\bar{s}$
Δ^{++}	uuu	K^0	$d\bar{s}$
Σ^+	uus	$\bar{K}^0$	$s\bar{d}$
Σ^0	uds	K^-	$s\bar{u}$
Σ^-	dds	J/ψ	$c\bar{c}$
Ξ^0	uss	D^+	$c\bar{d}$
Ξ^-	dss	D^0	$c\bar{u}$
Ω^-	sss	D_s^+	$c\bar{s}$
Λ_c^+	udc	B^+	$u\bar{b}$
Σ_c^{++}	uuc	$\bar{B}^0$	$\bar{d}b$
Σ_c^+	udc	B^0	$d\bar{b}$
Ξ_c^+	usc	B^-	$\bar{u}b$

Table 12-6 Some quantum numbers of the hadrons that are stable against decay via the strong interaction

Particle	Spin, $\hbar$	I	I_3	B	S	Y
p	1/2	1/2	+1/2	1	0	1
n	1/2	1/2	−1/2	1	0	1
Λ^0	1/2	0	0	1	−1	0
Σ^+	1/2	1	+1	1	−1	0
Σ^0	1/2	1	0	1	−1	0
Σ^-	1/2	1	−1	1	−1	0
Ξ^0	1/2	1/2	+1/2	1	−2	−1
Ξ^-	1/2	1/2	−1/2	1	−2	−1
Ω^-	3/2	0	0	1	−3	−2
π^+	0	1	+1	0	0	0
π^0	0	1	0	0	0	0
π^-	0	1	−1	0	0	0
K^+	0	1/2	+1/2	0	+1	+1
K^0	0	1/2	−1/2	0	+1	+1
η^0	0	0	0	0	0	0

Table 12-3 Hadrons that are stable against decay via the strong interaction

Name	Symbol	Mass (MeV/c^2)	Spin ($\hbar$)	Charge (e)	Antiparticle	Mean lifetime (s)	Typical decay products*
Baryons							
Nucleon	p (proton) or N^+	938.3	1/2	+1	$\bar{p}$	$>10^{32}$ y	
	n (neutron) or N^0	939.6	1/2	0	$\bar{n}$	930	$p + e^- + \bar{\nu}_e$
Lambda	Λ^0	1116	1/2	0	$\bar{\Lambda}^0$	2.5×10^{-10}	$p + \pi^-$
Sigma	Σ^+	1189	1/2	+1	$\bar{\Sigma}^-$	0.8×10^{-10}	$n + \pi^+$
	Σ^0	1192	1/2	0	$\bar{\Sigma}^0$	10^{-20}	$\Lambda^0 + \gamma$
	Σ^-	1197	1/2	−1	$\bar{\Sigma}^+$	1.7×10^{-10}	$n + \pi^-$
Xi†	Ξ^0	1315	1/2	0	$\bar{\Xi}^0$	3.0×10^{-10}	$\Lambda^0 + \pi^0$
	Ξ^-	1321	1/2	−1	$\bar{\Xi}^+$	1.7×10^{-10}	$\Lambda^0 + \pi^-$
Omega	Ω^-	1672	3/2	−1	Ω^+	1.3×10^{-10}	$\Xi^0 + \pi^-$
Charmed lambda	Λ_c^+	2285	1/2	+1	$\bar{\Lambda}_c^-$	1.8×10^{-13}	$p + K^- + \Lambda^+$
Mesons							
Pion	π^+	139.6	0	+1	π^-	2.6×10^{-8}	$\mu^+ + \nu_\mu$
	π^0	135	0	0	self	0.8×10^{-16}	$\gamma + \gamma$
	π^-	139.6	0	−1	π^+	2.6×10^{-8}	$\mu^- + \bar{\nu}_\mu$
Kaon	K^+	493.7	0	+1	K^-	1.24×10^{-8}	$\pi^+ + \pi^0$
	K^0	497.7	0	0	$\bar{K}^0$	0.88×10^{-10}	$\pi^+ + \pi^-$
						and	
						5.2×10^{-8}‡	$\pi^+ + e^- + \bar{\nu}_e$
Eta	η^0	549	0	0	self	2×10^{-19}	$\gamma + \gamma$

*Other decay modes also occur for most particles.
†The Ξ particle is sometimes called the cascade.
‡The K^0 has two distinct lifetimes, sometimes referred to as K^0_{short} and K^0_{long}. All other particles have a unique lifetime.

Lepton masses:
$m_{\text{electron}} = 0.511$ MeV$/c^2$
$m_{\text{muon}} = 105.7$ MeV$/c^2$
$m_{\text{tau}} = 1.777$ GeV$/c^2$

Delta resonance masses:
$m(\Delta^-) = m(\Delta^0) = m(\Delta^+) = m(\Delta^{++}) = 1232$ MeV$/c^2$

Delta resonance quark content:
$\Delta^-(ddd), \Delta^0(udd), \Delta^+(uud)$ and $\Delta^{++}(uuu)$