Structuur der Materie
exam 2019

9-4-2019

Abstract

Time: 13:30 – 16:30 (3 hours) – Please do not leave before 14:15.
The exam consists of two parts:
Part I tests knowledge in ten multiple-choice questions.
Part II consists of four open exercises.
The maximal number of points is indicated for each exercise.
Answer each of the exercises on a separate piece of paper.
Write your name and student number on each page.
Do not give final answers only, explain your reasoning (short) and give full calculations. A simple calculator use is allowed (not programmable).
No mobile/smart phone!
Good luck!

1 Part 1 - Multiple Choice questions (20 points)

1. The radius of a nucleus is:
   - A. between 10^{-12} m to 10^{-15} m
   - B. between 10^{-15} m to 10^{-14} m
   - C. between 10^{-10} m to10^{-12} m
   - D. between 10^{-10} m to 10^{-6} m
2. The main process by which energy is released in our Sun is called:
   - A. fission
   - B. Rutherford scattering
   - C. fusion
   - D. radioactivity
3. The rest energy of a proton is of the order of:
   - A. eV
   - B. keV
   - C. MeV
   - D. GeV
4. In a β^{+} decay an up quark becomes:
   - A. a strange quark
   - B. a down quark
   - C. an anti-quark
   - D. a top-quark
5. Most of the space in an atom is:
   - A. filled with neutrons
   - B. filled with negative charge
   - C. empty
   - D. filled with positive charge
6. α-particles have compared to other radiation relatively:
   - A. low kinetic energies
   - B. high potential energy
   - C. high mechanical energy
   - D. high kinetic energy
7. Particles such as the kaon and muon were found by:
   - A. looking at cosmic rays
   - B. using particle accelerators
   - C. studying the atom
   - D. both A and B
8. Which particles do not interact via the strong interaction?
   - A. protons

B. leptons

C. neutrons

D. gluons

9. Which of the following nuclei has the highest binding energy per nucleon?

A. $^{56}_{26}Fe$

B. $^{14}_{7}N$

C. $^{16}_{8}O$

D. $^{238}_{92}U$

10. The density of the proton is equal to the density of:

A. electron

B. atom

C. neutron

D. neutrino

2 Part 2 - Open questions

2.1 Nuclear Binding Energy (20 points)

The Semi-empirical mass formula is given by:

$$M(Z, A)c^2 = ZM_pc^2 + NM_nc^2 - E_b$$

Where M is the mass of the nucleus, A the sum of neutrons and protons, M_p the mass of the proton, M_n the mass of the neutron, Z the charge, N the number of neutrons, c the speed of light, and

$$E_b = a_1A - a_2A^{2/3} - a_3Z^2A^{-1/3} - a_4(A - 2Z)^2A^{-1} + a_5A^{-1/2},$$

is the binding energy.

a) (**7 points**) What do the terms a_1 to a_5 physically stand for (explain each term like $a_1A = ...$)

b) (**3 points**) The first term a_1A is the most important, what does this tell us about the nuclear force keeping the nucleons together?

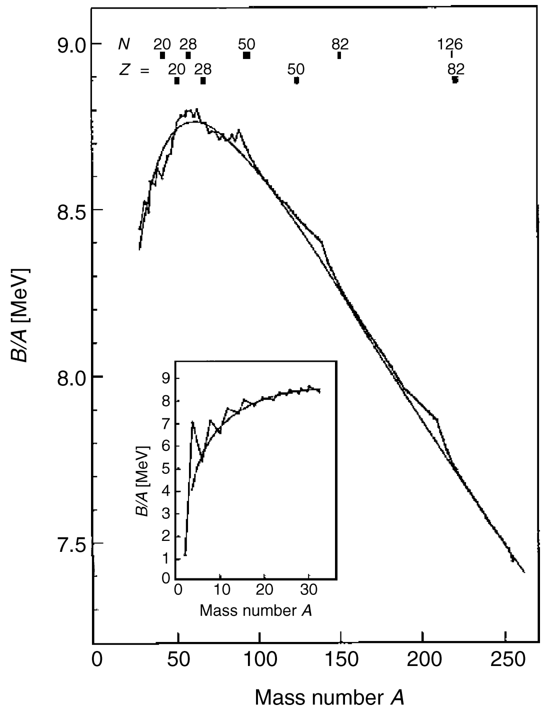


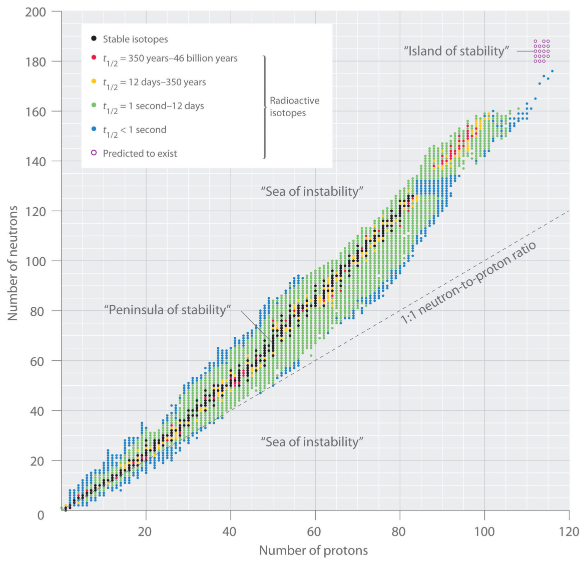
Figure 7.3 Binding energy per nucleon for even values of *A*: the solid curve is the SEMF (from Bo69)

c) (**5 points**) The figure shows the binding energy (B in this figure) per nucleon, explain why the spikes occur in the inset of the figure for different masses with an $A < 30$?

d) (**5 points**) Give the name of the model which describes these deviations and explain why and where this one also breaks down in describing the properties of a nucleus?

2.2 α and β-decay (20 points)

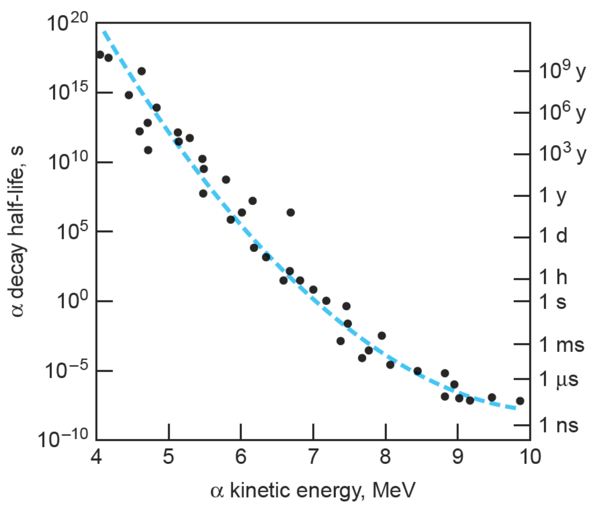
The figure below shows the various isotopes and their decay time.



a) (**6 points**) Describe where in this figure the isotopes decay via e^-, e^+, or α-particle decay

b) (**4 points**) What is the reason isotopes decay via β-decay?

For α-particle decay it is observed that there is a relation between the energy of the α-particle emitted and the decay-time (see figure below), the so-called Geiger-Nuttall rule.



c) (**5 points**) What is the physical reason for this relation as given by Gamov?

d) (**5 points**) Is the energy spectrum of β-decay continuous or discrete, and is α-decay continuous or discrete? Explain why.

2.3 Conservation laws and Feynman diagrams (20 points)

Check the following particle reactions and decays for violation of the conservation of energy/mass, electric charge, baryon number, lepton number and strangeness number (use the enclosed tables) say whether they are allowed or forbidden and why:

a) (**2 points**) $\Lambda^0 \rightarrow n + \gamma$

b) (**2 points**) $n + n \rightarrow p + p + e^- + e^-$

c) (**2 points**) $e^- + p \rightarrow n + \nu_e$

d) (**2 points**) $\Omega^- \rightarrow K^- + \Lambda^0$

e) (**2 points**) $p + p \rightarrow p + p + \bar{p} + \bar{p}$

f) (**2 points**) $J/\Psi \rightarrow \mu^+ + \mu^-$

Write down the Feynman diagrams on quark level for the following particle reactions (for the quark content of each particle see the enclosed tables):

g) (**4 points**) $\Delta^0 \rightarrow p + \pi^-$

h) (**4 points**) $K^0 \rightarrow \pi^+\pi^-$

2.4 Higgs production (20 points)

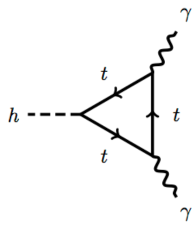
The Higgs particle has recently been discovered and has a mass of about 125 GeV/c^2.

$$p + p \rightarrow p + p + H$$

a) (**5 points**) Assume this reaction happens in a proton–proton collider experiment. What is the minimal energy the colliding protons should have for this reaction to occur?

b) (**5 points**) What should the energy of a proton be if it hits a stationary proton target (where one proton is at rest)?

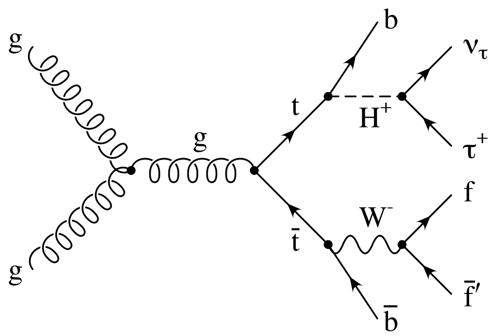
The Higgs particle is unstable and one possible decay is in two photons



c) (**5 points**) Explain how one can show that the Higgs particle is produced if one measured in a detector the reaction:

$$p + p \rightarrow p + p + H \rightarrow p + p + \gamma + \gamma$$

Now consider if a hypothetical positively charged Higgs, with also a mass of 125 GeV/c^2, is produced in the following reaction:



d) (**5 points**) What is the minimal energy in the center of mass of the two colliding gluons assuming that the H^+, the W^-, and the b and $\bar{b}$ are real particles?

Table 12-11 Quark composition of selected hadrons

Baryons	Quarks	Mesons	Quarks
p	uud	π^+	$u\bar{d}$
n	udd	π^-	$\bar{u}d$
Λ^0	uds	K^+	$u\bar{s}$
Δ^{++}	uuu	K^0	$d\bar{s}$
Σ^+	uus	$\bar{K}^0$	$s\bar{d}$
Σ^0	uds	K^-	$s\bar{u}$
Σ^-	dds	J/ψ	$c\bar{c}$
Ξ^0	uss	D^+	$c\bar{d}$
Ξ^-	dss	D^0	$c\bar{u}$
Ω^-	sss	D_s^+	$c\bar{s}$
Λ_c^+	udc	B^+	$u\bar{b}$
Σ_c^{++}	uuc	$\bar{B}^0$	$\bar{d}b$
Σ_c^+	udc	B^0	$d\bar{b}$
Ξ_c^+	usc	B^-	$\bar{u}b$

Table 12-6 Some quantum numbers of the hadrons that are stable against decay via the strong interaction

Particle	Spin, $\hbar$	I	I_3	B	S	Y
p	1/2	1/2	+1/2	1	0	1
n	1/2	1/2	−1/2	1	0	1
Λ^0	1/2	0	0	1	−1	0
Σ^+	1/2	1	+1	1	−1	0
Σ^0	1/2	1	0	1	−1	0
Σ^-	1/2	1	−1	1	−1	0
Ξ^0	1/2	1/2	+1/2	1	−2	−1
Ξ^-	1/2	1/2	−1/2	1	−2	−1
Ω^-	3/2	0	0	1	−3	−2
π^+	0	1	+1	0	0	0
π^0	0	1	0	0	0	0
π^-	0	1	−1	0	0	0
K^+	0	1/2	+1/2	0	+1	+1
K^0	0	1/2	−1/2	0	+1	+1
η^0	0	0	0	0	0	0

Table 12-3 Hadrons that are stable against decay via the strong interaction

Name	Symbol	Mass (MeV/c^2)	Spin ($\hbar$)	Charge (e)	Antiparticle	Mean lifetime (s)	Typical decay products*
Baryons							
Nucleon	p (proton) or N^+	938.3	1/2	+1	$\bar{p}$	$>10^{32}$ y	
	n (neutron) or N^0	939.6	1/2	0	$\bar{n}$	930	$p + e^- + \bar{\nu}_e$
Lambda	Λ^0	1116	1/2	0	$\bar{\Lambda}^0$	2.5×10^{-10}	$p + \pi^-$
Sigma	Σ^+	1189	1/2	+1	$\bar{\Sigma}^-$	0.8×10^{-10}	$n + \pi^+$
	Σ^0	1192	1/2	0	$\bar{\Sigma}^0$	10^{-20}	$\Lambda^0 + \gamma$
	Σ^-	1197	1/2	−1	$\bar{\Sigma}^+$	1.7×10^{-10}	$n + \pi^-$
Xi†	Ξ^0	1315	1/2	0	$\bar{\Xi}^0$	3.0×10^{-10}	$\Lambda^0 + \pi^0$
	Ξ^-	1321	1/2	−1	$\bar{\Xi}^+$	1.7×10^{-10}	$\Lambda^0 + \pi^-$
Omega	Ω^-	1672	3/2	−1	Ω^+	1.3×10^{-10}	$\Xi^0 + \pi^-$
Charmed lambda	Λ_c^+	2285	1/2	+1	$\bar{\Lambda}_c^-$	1.8×10^{-13}	$p + K^- + \Lambda^+$
Mesons							
Pion	π^+	139.6	0	+1	π^-	2.6×10^{-8}	$\mu^+ + \nu_\mu$
	π^0	135	0	0	self	0.8×10^{-16}	$\gamma + \gamma$
	π^-	139.6	0	−1	π^+	2.6×10^{-8}	$\mu^- + \bar{\nu}_\mu$
Kaon	K^+	493.7	0	+1	K^-	1.24×10^{-8}	$\pi^+ + \pi^0$
	K^0	497.7	0	0	$\bar{K}^0$	0.88×10^{-10} and 5.2×10^{-8}‡	$\pi^+ + \pi^-$ and $\pi^+ + e^- + \bar{\nu}_e$
Eta	η^0	549	0	0	self	2×10^{-19}	$\gamma + \gamma$

*Other decay modes also occur for most particles.
†The Ξ particle is sometimes called the cascade.
‡The K^0 has two distinct lifetimes, sometimes referred to as K^0_{short} and K^0_{long}. All other particles have a unique lifetime.

Lepton masses:
$m_{\text{electron}} = 0.511$ MeV$/c^2$
$m_{\text{muon}} = 105.7$ MeV$/c^2$
$m_{\text{tau}} = 1.777$ GeV$/c^2$

a few quark and boson masses:
mass W^- and $W^+ = 80.385$ GeV$/c^2$
mass $Z^0 = 91.2$ GeV$/c^2$
mass c and $\bar{c} = 1.28$ GeV$/c^2$
mass b and $\bar{b} = 4.2$ GeV$/c^2$
mass t and $\bar{t} = 173.2$ GeV$/c^2$

Delta resonance masses:
$m(\Delta^-) = m(\Delta^0) = m(\Delta^+) = m(\Delta^{++}) = 1232\ \text{MeV}/c^2$

Delta resonance quark content:
$\Delta^-(ddd), \Delta^0(udd), \Delta^+(uud)$ and $\Delta^{++}(uuu)$