Examination Exercises Particle Physics I

26 October 2011

Please remember to:

- make all exercises on seperate pieces of paper,
- add your name on each piece of paper you hand in,
- write in a readable way(!): it is better to make a readable exercise with mistakes than to make an unreadable exercise that is correct.

1: Theory Questions

(a) Electromagnetism

(i) Write (don't derive) the definitions of the $\vec{E}$ and $\vec{B}$ fields in terms of the vector potential A^μ.

(ii) The antisymmetric electromagnetic field tensor is defined as: $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. Use this to derive the components of the 4-by-4 field tensor in terms of the $\vec{E}$-field and $\vec{B}$-field components.

(b) Gauge Theory

(i) Which U(1), SU(2) and SU(3) local gauge invariances are implemented in nature according to the Standard Model? What are the related quantum numbers?

(ii) Give the expression for a SU(3) local gauge transformation acting on a quark spinor triplet wave-function. p 126

(c) Feynman Rules
Consider a scattering process of two incoming particles and two outgoing particles.

(i) Draw the t-channel diagram in which two incoming particles scatter in the field of a force carrier particle.

(ii) Give the Matrix element in the case that the scattering particles are spinless ($S = 0$) and that the force carrier is a photon.

(iii) Give the Matrix element in the case that the scattering particles are spinors and the force carrier is a W-boson.

2: Dirac Spinors and couplings

A positive energy Dirac spinor, i.e. a solution of the Dirac equation with positive energy, can be written as:

$$u^{(s)} = N \begin{pmatrix} \chi^{(s)} \\ \frac{\vec\sigma\cdot\vec p}{E+m}\,\chi^{(s)} \end{pmatrix} \quad \text{where} \quad \chi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \chi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The chirality and helicity operators are given by:

$$\text{chirality} = \gamma^5 \quad ; \quad \text{helicity} = \lambda = \frac{1}{2}\vec\Sigma\cdot\hat p \quad \text{with} \quad \vec\Sigma = \begin{pmatrix} \vec\sigma & 0 \\ 0 & \vec\sigma \end{pmatrix}$$

$\vec\Sigma\cdot\hat p \approx \gamma^5$

(a) Which quantum number is represented by the label s in the spinor?

(b) Give (*don't derive!*) γ^5 in the Dirac-Pauli representation.

(b) Decompose the Dirac spinor in left and right handed projection operators. Express your answer in general terms of γ^5 and u.

(c) Write the bilinear covariant ("vertex factor") for a *scalar interaction* and work it out in terms of right and left handed components u_R and u_L.

(d) Write the bilinear covariant ("vertex factor") for a *vector interaction* and work it out in terms of right and left handed components u_R and u_L

(e) Show that in the ultrarelativistic limit the chirality operator and the helicity operator have the same effect on the Dirac spinor, up to a factor 1/2.

(f) Compare the helicity behaviour of a vector interaction and a scalar interaction in the ultrarelativistic limit.

3: The Higgs Field

Part of the local symmetry group of the Standard Model is $SU(2)_T \otimes U(1)_Y$, where T stands for weak isospin, and Y stands for hypercharge. Consider the Higgs field, which transforms as a doublet under weak isospin, so we write it as

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{1}$$

The field Φ has hypercharge $+1$.

(a) Write down an infinitesimal, combined $SU(2)_T \otimes U(1)_Y$ transformation of Φ in terms of small parameters and group generators $T_i = \frac{1}{2}\tau_i$ and Y. Use that for small α the approximation holds: $\exp(\alpha) \approx 1 + \alpha$, and ignore second order terms.

In the groundstate of the Standard Model, we think the Φ field takes the value

$$\Phi_{\text{vac}} = \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{2}$$

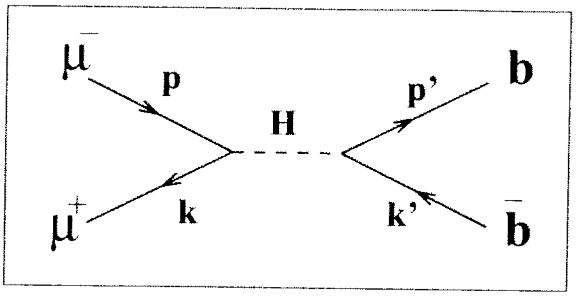
where $v \approx 246$ GeV is a constant.

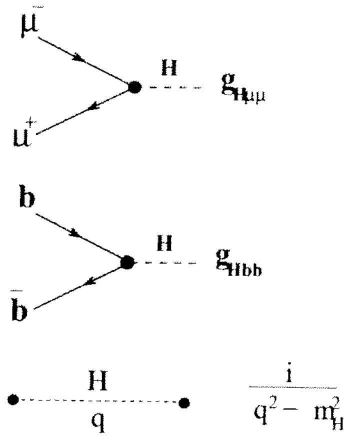
(b) Write out the transformation of the groundstate Φ_{vac} and show that only the symmetry generated by the particular combination $T_3 + \frac{1}{2}Y$ leaves this groundstate invariant under the infinitesimal transformation.

The Pauli matrices are:

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad ; \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad ; \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

4: The process $\mu^-\mu^+ \to H \to b\bar{b}$





(a) Give the lowest order Feynman amplitude (i.e. the Matrix element) for the $\mu^+\mu^- \to H \to b\bar{b}$ process corresponding to the diagram shown in the box. Note that the 4-momenta p,k are the momenta of the incoming μ^- and μ^+ spinors, the 4-momenta p' and k' are the momenta of the outgoing b and $\bar{b}$ spinors. The Feynman rules which are new to you, the Higgs propagator and the Higgs coupling constants, are given above on the right.) The Higgs has S=0, i.e. it is a scalar interaction!

(b) Show that the spin averaged and summed amplitude square reduces, after the Trace calculation and after neglecting muon and b-quark mass effects, to:

$$\overline{|M|}^2 = g^2_{H\mu\mu}\, g^2_{Hbb}\, \frac{s^2}{|s - m_H^2|^2} \tag{3}$$

where m_H is the Higgs mass.

(c) Calculate the differential cross section $d\sigma/d\Omega$. Show that the total $\mu^-\mu^+ \to H \to b\bar{b}$ cross section can be written as:

$$\sigma(\mu^-\mu^+ \to H \to b\bar{b}) = g^2_{H\mu\mu}\, g^2_{Hbb}\, \frac{1}{16\pi}\, \frac{s}{|s - m_H^2|^2} \tag{4}$$

(d) To take into account the fact that the Higgs is an unstable particle the above given propagator for the Higgs particle must be changed as follows (Γ_H represents the total decay width of the Higgs particle):

$$\frac{i}{q^2 - m_H^2} \longrightarrow \frac{i}{q^2 - m_H^2 + i m_H \Gamma_H} \tag{5}$$

Use this changed Higgs propagator to sketch how the total cross section depends on the c.m. energy $\sqrt(s)$ of the $\mu^-\mu^+$ system. Also explain how from measurements in a certain c.m. energy range one can determine the Higgs mass, m_H, and the Higgs total decay width, Γ_H. Which combination of reactions do you need to investigate in order to seperately determine g_{Hbb} and $g_{H\mu\mu}$?