# Test Exam (Re-exam 2017/2018)

You have two hours to complete this exam.

Write your name and student number on each sheet of paper that you hand in. Write the total number of sheets on the first sheet.

Give concise answers in clear Dutch or English. Write clearly. When not asked explicitly, you may directly use results discussed in class; mention this explicitly when you do.

If you are asked for a formula or calculation, make sure you explain your intermediate steps for possible partial credit.

The exam has five exercises. Make sure to allocate your time well; some exercises may be harder than others.

### 1. Erdős-Rényi ($2 = 1 + 0.5 + 0.5$ points)

Consider the Erdős-Rényi random graph model.

(a) Give the two main reasons why this model does not accurately capture most real networks. Explain your answers.

(b) What is (roughly) the expected size of the largest component at the critical point of the model?

(c) Consider a graph generated from this model such that $N = 5000$ and $p = 10^{-5}$. In which regime is the graph? *The correct answer yields 0.5 points; a wrong answer yields -0.25 points; no answer yields 0 points.*

- A Supercritical regime.
- B Anomalous regime.
- C Subcritical regime.
- D Ultra-Small World regime.

### 2. Clustering coefficient ($2.25 = 1 + 0.5 + 0.75$ points)

(a) Define and explain in detail, the global clustering coefficient and the average local clustering coefficient of a graph.

(b) Construct, for any $N$, an example of a graph of $N$ vertices where these coefficients differ.

(c) Consider a graph where each vertex has degree 2 and the average local clustering coefficient is 1. What does this graph look like? Explain your answer.

**3. Configuration model** ($2.25 = 0.5 + 0.75 + 0.75$ **points**)

Consider a graph generated by the configuration model.

(a) State the Molloy-Reed criterion for the existence of a giant component in a graph.

(b) Suppose that the degree distribution $p_k$ satisfies $p_0 = 0$ and $p_k = 0$ for $k > 3$; that is, only $p_1, p_2, p_3$ are potentially bigger than 0. Argue that the graph has a giant component if $p_1 < 3p_3$.

(c) Suppose the degree sequence used to generate the configuration model graph specifies degree $k_i$ for vertex $i$. Derive a simple expression for the expected number of common neighbors of two vertices $i$ and $j$.

**4. Network growth** ($1.75 = 0.5 + 0.5 + 0.75$ **points**)

Preferential attachment is a model for network growth. A vertex joining at time $i$ has degree $k_i(t)$ at time $t$. Recall that a new vertex joining at time $t$ connects to $m$ existing vertices; it connects to the vertex that joined at time $i < t$ with probability proportional to $k_i(t)$.

(a) Give the formula for $k_i(t)$, the degree of vertex $i$ at time $t$, as a function of $t$.

(b) Suppose the new vertex connects to an existing vertex with probability proportional to $(k_i(t))^3$. Describe the expected structure of the graph.

(c) Name three other models of network growth discussed in class.

**5. Robustness** ($1.75 = 0.5 + 0.5 + 0.75$ **points**)

(a) Consider a very large scale-free graph with power-law exponent $2 < \gamma < 3$. An attacker disables 80% of the vertices of the graph at random. Does this fragment the graph into many small components? Explain your answer.

(b) Consider a very large scale-free graph with a large power-law exponent $\gamma$. An attacker disables the 80% *highest-degree* vertices of the graph. Does this fragment the graph into many small components? Explain your answer.

(c) Describe the structure of graphs that have the highest tolerance against both random and targeted attacks (i.e. the removal of high-degree vertices).