

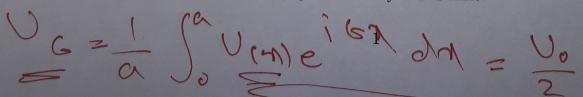
- (a) Sketch the shape of the potential for the electrons and holes.
- (b) What is the effect of the electron confinement in the potential well on the minimum energy of the conduction band. (Hint: expand the parabolic dispersion of electrons in terms the 3 momentum components and consider the effect of the potential well on each of the components).
- (c) What approximate value of L is required if the band gap of the quantum well is to be 0.1 eV larger than that of GaAs bulk material?

3. Vibrations in a harmonic chain with next-nearest-neighbour coupling Consider the lattice vibrations of a chain of identical masses m, in which each mass is connected to its first and second nearest neighbours by springs of spring constant K_1 and K_2 respectively, and the equilibrium spacing between neighboring masses is a. Show that

(a) the lattice vibrations follow the dispersion relation

$$m\omega^2 = 2K_1[1-\cos(ka)] + 2K_2[1-\cos(2ka)]$$

(b) the dispersion relation in (a) reduces to that for sound waves in the long wavelength limit. What is the propagation velocity in this limit?



The magnetization M is given by

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$$M = N \frac{\sum_{M_J=-J}^{+J} (-g_J \mu_B M_J) f(M_J)}{\sum_{M_J=-J}^{+J} f(M_J)},$$

with N the atomic density and the Boltzmann factor $f(M_J) = \exp(-g_j \mu_B M_J B/(k_B T))$.

- (h) Explain in physical terms the Boltzmann factor.
- (i) Show that for large temperature that the susceptibility $\chi \equiv \mu_0 M/B$ is proportional to 1/T.
- (j) Under what condition is the result in the previous item valid?

4. Different kinds of magnetism

- (a) Why is Fe^{3+} above the Curie temperature a paramagnet? Provide a physical reasoning.
- (b) Why does $\mathrm{Fe^{3+}}$ become a ferromagnet below the Curie temperature? Provide a physical reasoning.

A ferromagnet below the Curie temperature can be described by the Heisenberg Hamiltonian:

$$\mathcal{H} = -J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j,$$

where the sum runs over only the nearest neighbors.

- (c) Why do we neglect in the sum all the other atoms apart from the neareast neighbors?
- (d) What is the microscopic nature of the interaction term J? Spin waves can be excited in ferromagnets.
- (e) Can we also excite spin waves above the Curie temperature? Explain your answer.
- (f) Can spin waves also be induced in antiferromagnets? Explain your answer.
- (g) How can spin waves be used for logic gates?