

Three-dimensional Modeling (INFODDM) April 20, 2010

Part 1

In this part, all questions weigh equally.

Opgave 1. Acquisition

- Describe how triangulation works to compute the coordinates of a point on the surface, when scanned with a laser scanner.
- What is the difference between active and passive optical acquisition methods?
- What is the advantage of a non-contact way of acquisition?

Opgave 2. Alignment

- Describe how to align a point set with the z-axis.
- Suppose we have the following points: $(2, -4), (3, -4), (3, -2), (5, -1), (5, 1), (6, -2), (4, -3), (4, -1)$. Compute the eigenvalues of the covariance matrix.
- What are the first and second principal components of a two-dimensional point set centered around the origin, with $cov(x, x) = 2, cov(y, y) = 2$, and $cov(x, y) = -5$?

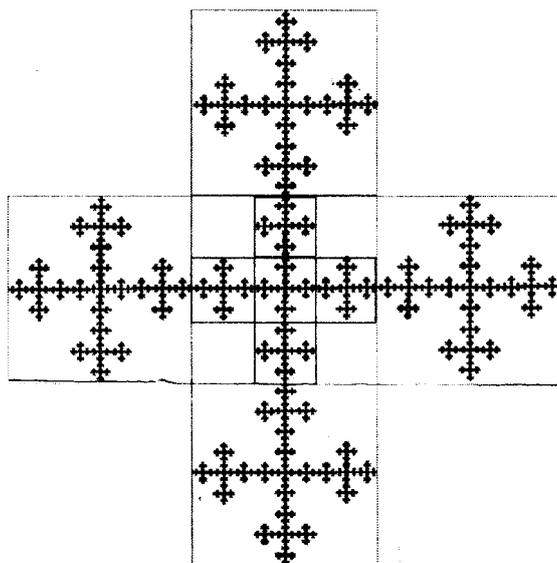
Opgave 3. Simplification

The following questions are related to Garland and Heckbert's surface simplification method (from the paper Surface Simplification using Quadric Error Metrics).

- What error measure does the matrix Q of a vertex store?
- When contracting two vertices the error measure matrix Q for the new vertex is created by simply taking the sum of the two matrices Q_1 and Q_2 of the two vertices that are contracted. Describe why this simple summation of the matrices is allowed, and explain how this could cause an approximation error of the contraction target?
- The method does not preserve topology. However, there are methods for surface simplification that do. Give one example why the non-conservation of topology would be beneficial, and explain the reason for this.

Opgave 4. Procedural modeling

- a) What is the definition of a topological dimension and fractal dimension of a curve?
- b) What is affine self-similarity of a point set?
- c) The Vicsek fractal, also known as Vicsek snowflake or box fractal, arises from a construction similar to that of the Sierpinski carpet. The filled basic square is decomposed into nine smaller filled squares in the 3-by-3 grid. The four squares at the corners are removed, leaving the five squares that form a '+'-sign. The process is repeated recursively for each of the five remaining sub squares (see the figure below). Determine the fractal dimension of this fractal, and motivate your answer.



Part 2

Total maximum score for this part: 90 points.

Opgave Curves and surfaces

1. (15 points) A NURBS-curve has three types of parameters controlling the shape of the curve. Name each type, give the effect of each type on the shape of the curve, and explain briefly (using a mathematical model or a description of it) how each effect is achieved.
2. (10 points) Give an equation of the tangent line to $\mathbf{Q}(t) = \left(\frac{t^2}{4}, \sin(t)\right)$ at $t = 0$.
3. (10 points) When computing a surface – represented as a spline surface – by sweeping a spline curve through 3D space, what extra problem is encountered when using non-interpolating splines instead of interpolating splines?
4. (10 points) Describe how 2D free-form deformation works and give a general formula.
5. (10 points) Compute the normal vector of the ellipsoid $(\frac{x}{2})^2 + y^2 + z^2 = 1$ at the point $(2, 0, 0)$.

Opgave Animation

6. (15 points) Which object model would you use for (and say why):
 - a) Smoke
 - b) A shaking cube of gelatin
 - c) A goat
 - d) A school of fish
 - e) A long human hair
7. (20 points)
 - a) Give the quaternion p that is associated with rotating an object (in 3D space) around the y -axis by $\pi/4$ radians.
 - b) Given a second quaternion $q = (0, (0, 0, 1))$, give the quaternion r that is associated with the compound rotation achieved by first applying the rotation associated with p , and then the rotation associated with q . (Reminder: $qq' = (ss' - v \cdot v', v \times v' + sv' + s'v)$.)