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In 2009-2010, the course INFODDM was given by Prof. Dr. R.C.Veltkamp.

3D Modeling (INFODDM)
25 May 2010

It is not allowed to consult books, notes, telephone etc., or someone else's answers.
Always explain your answer, used symbols, etc.
Be precise.

Part 1

All questions in the first part weight equal.

Question 1. Acquisition

A laser range scanner shines a laser on the subject and exploits a camera to look for the location of the laser dot.

a) Describe how *triangulation* is used to compute the 3D coordinates of the dot.

b) *Structured light* can be used to speed up the scanning process. Describe how this technique works.

c) Give three problems which have to be addressed when a surface is acquired. Give a short explanation for each problem.

Question 2. Alignment

The paper *Efficient Variants of the ICP Algorithm* by Rusinkiewicz and Levoy describes different variations on the *Iterative Closest Point (ICP)* algorithm. This algorithm tries to align two points sets in the best possible manner.

a) Explain the basics of the *Iterative Closest Point* algorithm, i.e. how does it work globally?

b) Variants of the ICP can be classified as affecting one of six stages of algorithm. Name two of these stages, and for every stage give at least one example of what could be changed.

Question 3. Model Reconstruction

In one of the first papers on model reconstruction of general shapes, namely *Surface Reconstruction from Unorganized Points* by Hugues Hoppe et al. in 1992, a technique is introduced that uses a concept known as the *signed distance function.*

a) Explain what the *signed distance function* is and how it can be used to reconstruct a surface.

b) A number of assumptions on the shape (that needs to be reconstructed) need to be made in order for the approach to work. For example, the surface needs to be compact (finite triangulation). Give two additional assumptions and explain why these are necessary.

c) It is of paramount importance that the orientation of the tangent planes is consistent over the whole object. This is a very difficult problem for which an approximation technique is given that used the *Euclidian Minimal Spanning* Tree (MST). Explain how this approximation technique works.

Question 4. Fractals

Consider the following variation of the Koch snowflake:

- **F**: move forward 1 unit
- +: turn counter-clockwise by 90 degrees
- -: turn clockwise by 90 degrees
- production rule: $\mathbf{F} \rightarrow \mathbf{F}+\mathbf{F}-\mathbf{F}-\mathbf{FF}+\mathbf{F}+\mathbf{F}-\mathbf{F}$

Generation 0 is the string **F**.

a) Apply the production rule twice, and draw the resulting curve.

b) Compute the fractal dimension of the curve produced by production rule **F**.

c) Is the fractal dimension of this curve higher or lower than that of the original Koch snowflake? Why?

Part 2

In this part, the total maximum score is 90 points

Curves and surfaces

Question 1. (15p.)

a) Draw $\mathbf{Q}(t) = \left(\sin(t), \left(\frac{t}{\pi}\right)^2\right)$ for $t \in [-\pi, \pi]$

b) For what t is the tangent vector to $\mathbf{Q}$ horizontal?

Question 2. (10p.)

Compute $\mathbf{Q}_{uv}$, for $u = v = 1$ for a cubic Bézier patch if $\mathbf{P}$ is a control point matrix,

$$\mathbf{Q}(u, v) = \left[u^3 u^2 u 1\right) \mathbf{B}_z \mathbf{P} \mathbf{P}_z^T \begin{bmatrix} v^3 \\ v^2 \\ v \\ 1 \end{bmatrix}$$

and

$$\mathbf{B}_z = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

Question 3. (10p.)

Many curves $\mathbf{Q}$ are formulated as weighted combinations of a control points set, i.e.,

$$\mathbf{Q}(u) = \sum_i \mathbf{P}_i B_i(u),$$

with control points $\mathbf{P}_i$, curve parameter u and weight functions B_i.

a) Give a formula for the rational variant of this $\mathbf{Q}$, and

b) explain why the rational form is more flexible, i.e., can represent more curve shape variation than the non-rational form.

Question 4. (10p.)

Given a patch surface $\mathbf{Q}(u, v)$, give a general formula to compute the surface normal at (u, v).

Animation

Question 5. (15p.)

Which object model would you use for (and say why):

a) Smoke

b) A shaking cube of gelatin

c) A goat

d) A school of fish

e) A long human hair

Question 6. (20p.)

a) Give the quaternion p that is associated with rotating an object (in 3D space) around the y-axis by $\pi/4$ radians.

b) Given a second quatemion $q = (0, (0, 0, 1))$, give the quaternion r that is associated with the compound rotation achieved by first applying the rotation associated with p, and then the rotation associated with q. (Reminder: $qq' = (ss' - v \cdot v', v \times v' + sv' + s'v)$.)

Question 7. (10p.)

Explain why the *slerp* function is necessary to interpolate quaternions in an animated sequence.