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In 2005/2006, the course INFODDM was given by Twan Maintz.

3D Modeling (INFODDM)
April 20, 2006

Surface Simplification

In *Surface simplification using quadratic error metrics*, Garland and Heckbert describe how for a vertex v an error $\Delta(v)$ is defined as

$$\Delta(v) = \sum_{p \in \text{planes}(v)} (p^T v)^2$$

1. What is "planes(v)" for a vertex v of the initial (i.e., unsimplified) mesh, and what is the value of $\Delta(v)$ for such a vertex?

2. Vertices v of the mesh have a 4×4 matrix Q associated with them, representing "planes(v)". When a vertex v_k results from contracting a vertex pair v_i and v_j, the matrix Q_k is computed as $Q_i + Q_j$.

   a) Explain why a single plane can be counted multiple times in Q_k.

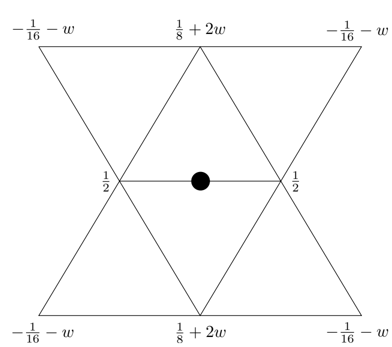
   b) How often can a single plane be counted at most, and why?

3. What is the difference between *edge contraction* and *pair contraction*, and what is the advantage of pair contraction over edge contraction?

Subdivision surfaces

4. In the Catmull-Clark subdivision scheme, several types of new vertices/points are distinguished in a refinement step. Describe algorithmically for each of the types of new points how they are created. (You may ignore extraordinary points.)

5. The figure below shows the mask for the Butterfly subdivision scheme. What is the role of the parameter w?



6. When is a subdivision scheme called *interpolating*? Is the Catmull-Clark scheme interpolating? And the Butterfly scheme?

Curves and surfaces

7. a) Draw $\mathbf{Q}(t) = (\frac{1}{4}t^2, (t-1)^2), t \in [0, 2]$.
   b) Give the tangent vector to $\mathbf{Q}$ for $t = 1$.

8. Show that a cubic Bézier curve (see formulas below) is tangent to its control polygon at the start and end point.

$$\begin{aligned}
\mathbf{Q}(u) &= \sum_{i=0}^{3} \mathbf{P}_i B_i(u) \\
B_0(u) &= (1-u)^3 \\
B_1(u) &= 3u(1-u)^2 \\
B_2(u) &= 3u^2(1-u) \\
B_3(u) &= u^3 \qquad u \in [0, 1]
\end{aligned}$$

9. Describe what happens to a B-spline curve when an affine transformation is applied to its control points.

10. Given a patch surface $\mathbf{Q}(u, v)$, give a general formula to compute the surface normal at (u, v).

Animation

11. Explain how a relatively simple modification of particle system animation can be used to animate flocking behaviour in animals.

12. Show by explicit multiplication that the formula for $qq' = (ss' - \mathbf{v} \cdot \mathbf{v}', \mathbf{v} \times \mathbf{v}' + s\mathbf{v}' + s'\mathbf{v})$ is correct for

$$\begin{aligned}
q = (s, \mathbf{v}) &= (2, (1, 1, 1)) \\
q' = (s', \mathbf{v}') &= (0, (2, 0, 2))
\end{aligned}$$

13. Explain why the *slerp* function is necessary to interpolate quaternions in an animated sequence.