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In 2006/2007, the course INFODDM was given by Twan Maintz & Roland Geraerts.

3D Modelling (INFODDM)
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Alignment

Question 1

a) Let $\boldsymbol{P} = \{(-2, 0), (-1, 1), (0, 2)\}$ be a point cloud. Compute the covariance matrix $\boldsymbol{C}$ of $\boldsymbol{P}$ as used in the Principal Component Analysis method for alignment.

b) The eigenvalues of matrix $\boldsymbol{C}$ can be computed by solving $\det(\boldsymbol{C}-\lambda\boldsymbol{I}) = 0$. Compute the two eigenvalues λ.

c) The eigenvectors of matrix C can be computed by solving $(C-\lambda\boldsymbol{I})\boldsymbol{\mu} = 0$. Compute the two eigenvectors $\boldsymbol{\mu}$.

d) Give the combined rotation and transformation matrix that aligns the point set with the principal axes.

e) If $\boldsymbol{\mu}$ is an eigenvector for $\boldsymbol{C}$, then $\boldsymbol{C\mu} = \lambda\boldsymbol{\mu}$. This implies that $-\boldsymbol{\mu}$ is also an eigenvector. What impact does this have on the transformation matrix from d)? What effect will negating $\boldsymbol{\mu}$ have on the transformed point cloud?

Simplification

Question 2

The 3D scanning pipeline consists of four stages to obtain a final model:

- surface points have to be gathered,
- partial surfaces have to be aligned,
- the aligned surface has to be reconstructed,
- the surface has to be simplified.

In each stage, many difficulties have to be tackled. Give four difficulties encountered in the simplification phase.

Terrains, Fractals and Procedural modelling

Question 3

a) Name three advantages of using the triangle as the building block of surface representation instead of more complex structures such as quadrilaterals.

b) What is the difference between the geometrical and topological information in a triangular mesh?

Question 4

Consider the following variation of the Koch snowflake:

- **F**: move forward 1 unit
- +: turn counter-clockwise by 90 degrees
- –: turn clockwise by 90 degrees
- production rule: **F→F+F–F–F+F**

Generation 0 is the string **F**.

a) Apply the production rule once, and draw the resulting curve.

b) Apply the production rule again, and draw the resulting curve.

c) Compute the fractal dimension of the curve produced by production rule **F**.

d) Is the fractal dimension of this curve higher or lower than that of the original Koch snowflake? Why?

Curves and Surfaces

Question 5

a) Draw the curve $\mathbf{Q}(t) = \left(\frac{1}{4}t^2, (t-1)^2\right),\ t \in [0, 2]$.

b) Give the tangent vector to $\mathbf{Q}$ for $t = 1$.

Question 6

Show that a cubic Bézier curve (see formulas below) is tangent to its control polygon at the start and end point.

$$\begin{aligned}
\mathbf{Q}(u) &= \sum_{i=0}^{3} \mathbf{P}_i B_i(u) \\
B_0(u) &= (1-u)^3 \\
B_1(u) &= 3u(1-u)^2 \\
B_2(u) &= 3u^2(1-u) \\
B_3(u) &= u^3 \qquad\qquad u \in [0, 1]
\end{aligned}$$

Question 7

Many curves $\mathbf{Q}$ are formulated as weighted combinations of a control points set, i.e.,

$$\mathbf{Q}(u) = \sum_i \mathbf{P}_i B_i(u),$$

with control points P_i, curve parameter u and weight functions B.

a) Give a formula for the rational variant of this $\mathbf{Q}$, and

b) explain why the rational form is more flexible, i.e., can represent more curve shape variation than the non-rational form.

Animation

Question 8

a) Compute the rotation angle θ and unit rotation axis $\mathbf{n}$ corresponding to the quaternion $q = \left(\frac{1}{2}\sqrt{2}, (\frac{1}{2}, 0, -\frac{1}{2})\right)$.

b) Compute the quaternion w corresponding to the rotation axis $\mathbf{n}$ and rotation angle $\frac{4}{3}\theta$.

c) First rotating an object by using the quaternion $\mathbf{Q}$ (in the standard way), then rotating the object further by using w results in a total rotation that can also be achieved by using a single quaternion p. Give an expression for p in terms of q and w. (You don't need to compute p).

Question 9

When using inverse kinematics for producing an animation, we may use forward (FK) or inverse kinematics (IK).

a) Give an advantage of using FK over IK.

b) Give an advantage of using IK over FK.