

## 3D Modelling (INFODDM)

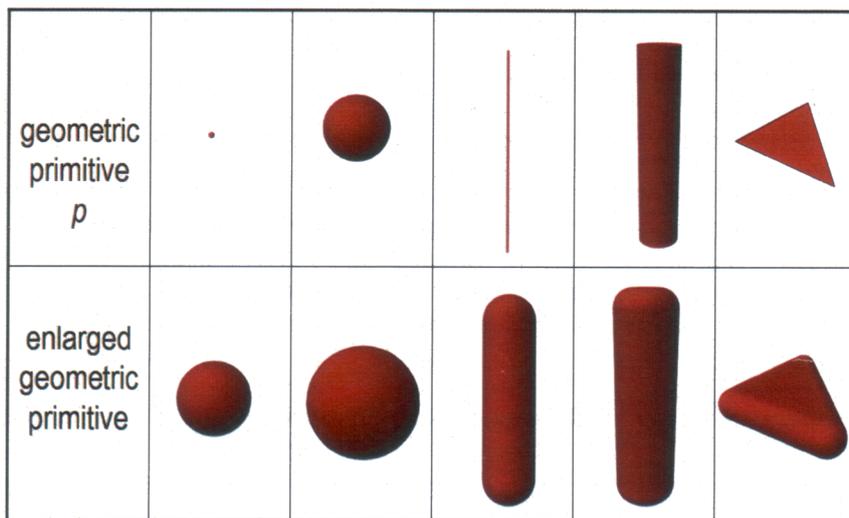
### April 14, 2008

## Introduction and representations

### Question 1

As part of a Constructive Solid Geometry toolkit, we would like to create an algorithm for enlarging a solid object. We assume that this object can be decomposed using a set of geometric primitives, referred to as  $P$ . In our algorithm, for each geometric primitive  $p \in P$ , we define a set of geometric primitives whose union represents the enlarged version of  $p$ . The enlarged solid object can then be represented as the union of the enlarged geometric primitives.

The figure below shows renderings of the following geometric primitives, together with their enlarged versions: point, sphere, line, cylinder and triangle.



- a) In theory, a geometric primitive can be enlarged by sweeping a sphere along each point on the primitive's surface. The enlarged version can then be represented as the union of all spheres. Why is this not an efficient algorithm?
- b) As shown in the figure, a point can be enlarged by replacing it with a sphere and a sphere can be enlarged by replacing it with a larger sphere. Give an efficient representation for an enlarged line. In this representation, give the list of primitives used and mention for each primitive how many you need. You have to choose from the following primitives: sphere, cylinder, torus, tetrahedron and cube.
- c) Give an efficient representation for an enlarged cylinder.
- d) Give an efficient representation for an enlarged triangle.

## Reconstruction

### Question 2

Hoppe's paper on *Surface reconstruction from Unorganized Points* describes a general method for surface reconstruction.

Given a noiseless set of points  $P = \{(-1, 0), (0, -1), (1, -2), (2, -1), (3, 0)\}$  and a point  $p = (\frac{1}{2}, -1\frac{1}{2})$  for which we want to compute the signed distance (to the appropriate tangent line). The parameter  $k$ , which denotes the number of nearest neighbors being considered in the computation for the tangent line, is set to 2.

- Draw the point set  $P$ .
- Give the two points from the set  $P$  which determine the tangent line for point  $p$ .
- Compute the center  $o$  of the tangent line. Next, compute a normalized normal  $\vec{n}$  of the tangent line by using *Principle Components Analysis*.
- Compute the signed distance of point  $p$  to the tangent line from c) by projecting  $p$  onto  $\vec{n}$ .

## Simplification & level of detail

### Question 3

The two following questions are related to Garland and Heckbert's surface simplification method (from the paper *Surface Simplification using Quadric Error Metrics*).

- What error measure does the matrix  $Q$  of a vertex store?
- What is the cause of the approximation error of the contraction target?

### Question 4

Give three reasons why you would want to store an object at multiple levels of detail.

## Subdivision surfaces

### Question 5

- Give three advantages of using subdivision surfaces compared to using a continuous representation such as NURBS patches.
- Is the Butterfly subdivision method *interpolating* or *approximating*? Explain.

## Curves and surfaces

### Question 6

A NURBS—curve has three types of parameters controlling the shape of the curve. Name each type, give the effect of each type on the shape of the curve, and explain briefly (using a mathematical model or a description of it) how each effect is achieved.

### Question 7

Give an equation of the tangent line to  $Q(t) = \left(\frac{t^2}{4} \sin(t)\right)$  at  $t = 0$ .

### Question 8

When computing a surface – represented as a spline surface – by sweeping a spline curve through 3D space, what extra problem is encountered when using non-interpolating splines instead of interpolating splines?

### Question 9

Describe how 2D free-form deformation works and give a general formula.

### Question 10

Compute the normal vector of the ellipsoid  $(\frac{x}{2})^2 + y^2 + z^2 = 1$  at the point  $(2, 0, 0)$ .

## 1 Animation

### Question 11

Which object model would you use for (and say why):

- a) Smoke
- b) A shaking cube of gelatin
- c) A goat
- d) A school of fish
- e) A long human hair

### Question 12

- a) Give the quaternion  $p$  that is associated with rotating an object (in 3D space) around the  $y$ -axis by  $\pi/4$  radians.
- b) Given a second quaternion  $q = (0, (0, 0, 1))$ , give the quaternion  $r$  that is associated with the compound rotation achieved by first applying the rotation associated with  $p$ , and then the rotation associated with  $q$ . (Reminder:  $qq' = (ss'|v \cdot v', v \times v' + sv' + s'v)$ .)